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ཏོག་ལས་ཁུངས།

Royal Government of Bhutan
Ministry of Agriculture and Forests
Department of Forests and Park Services

SPECIES SPECIFIC VOLUME EQUATION TO ESTIMATE MERCHANTABLE VOLUME

Pinus wallichiana

Forest Resources Management Division
Department of Forest and Park Services
Ministry of Agriculture and Forests
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Species specific volume equation to estimate merchantable volume

Pinus wallichiana

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1. Summary

The volume equation developed through this study will predict the merchantable volume of *Pinus wallichiana*. The merchantability standard for volume calculation adopted for this study is 10 cm and above diameter at breast height (dbh) and top diameter measured up to 10 cm over bark.

A total of 16 models were fitted. First 4 models were fitted with volume as a function of diameter at breast height (DBH), while models 5 – 8 were fitted with basal area (BA) as the predictor variable. With product of squared diameter at breast height and height (DBH2H) as predictor variable, 4 models, namely the models 9 – 12 were fitted. The last four models, 13 -16 were fitted with product of basal area and height (BAH) as the predictor.

The initial plots of response variable (volume) and predictor variables (DBH, BA, DBH2H and BAH) clearly indicated presence of heteroscedasticity, which has been modeled using variance functions (varFixed, varPower and varConstPower) in gls ( ) function of nlme package.

Of the sixteen, two models viz model 7 (fitted without height as predictor) and model 16 (fitted with height as predictor) with lower values of AIC and BIC have been selected as the best fit models for *Pinus wallichiana*. The model 7 had AIC and BIC values of -98 and -81 respectively, while the model 16 had AIC and BIC values of -414 and -393 respectively. Lower the AIC and BIC values, better the fit of the model.

The performance of the selected models was assessed by comparing the actual volume with the volumes predicted by two selected models for each tree. From the assessment, the model 16 which uses height outperforms the model 7.

2. Introduction

The volume equations, developed during pre-investment survey (PIS) carried out between 1974-81 predict total tree volume, and not the merchantable volume of trees. The recent change of policy of the Department of Forests and Park Services to allot timber for rural house construction in the form of log volume instead of allotting by number of trees as was once practiced, has necessitated development of merchantable log volume equation.

Therefore, standards of merchantability adopted for this study to develop merchantable log volume equation are trees of 10 cm and above diameter at breast height (dbh) and the sections up to 10 cm top diameter over the bark.

As was done for PIS exercise to develop volume equation, this study ignores/does not consider the volume of foliage and branches for the purpose of calculating the merchantable volume. This decision stems from the objective, which is to estimate merchantable log volume. Moreover, branches are rarely used as timber (at least in Bhutan) and are mostly used for firewood.

The sample trees for this study have been felled as part of biomass equation development field work. 8 trees each have been felled from four regions of Bhutan namely, eastern, eastern central, western and western central. In order to improve the model, additional samples were collected from forest management units (FMUs) and outside FMU areas within Paro, Thimphu and Haa Dzongkhags. In total 249 sample trees have been used for this modeling study.

The trees were felled at 0.3 m height from the ground at which the diameter was measured and recorded. After felling diameter was measured at 0.7 m from 0.3 m height (essentially making 1 m height, i.e 0.3 m + 0.7 m =1 m). Thereafter, at every meter length, the diameter was measured and recorded, thus making many 1 m length sections of log. As mentioned above the smallest top diameter considered for merchantable log volume calculation was up to 10 cm diameter over bark. Top sections below 10 cm diameter have been discarded.

3. Volume Calculation

Trees after felling are converted into different sizes of sections depending on the requirement and demand. Sections with length of 8 or more feet long are called logs and shorter ones are called sticks or bolts (Avery and Burkhart, 1994). The scaling or measuring the volume of the section is done by multiplying the length with the cross-sectional area of the section. Although they rarely form true circles, they are assumed so for the purpose of calculating cross sectional area in meter square, which is

$$\text{Cross sectional area (A)} = A = \pi r^2 = \frac{\pi D^2}{4*10000} \quad (1)$$

Where **r** is radius in meters and **D** is diameter at breast height in centimeters.

From the ground level to 0.3 m height (height at which sample tree has been cut) is section I, while 0.3 m to 0.7 m is section II. The subsequent sections of 1 m length each are numbered III, IV and so on. The last section is the terminal section, whose length is equal to or less than 1 m.

The diameter at zero height (ground level) for stump wasn't measured in the field (for those sample trees for which volume data was collected during biomass equation development field work) and therefore, calculated based on diameter reading at 0.3 m height. But for the additional samples collected from Paro, Thimphu and Haa, measurements were made in the field itself at 0 m height. Therefore, diameter at zero height (for those samples that didn't have diameter measurement at 0 height taken in the field) was calculated as 10% more than diameter at 0.3 m height, which is;

$$D_{(ground)} = D_{(0.3\ m)} + 10\% * D_{(0.3\ m)} \quad (2)$$

Where;

$D_{(ground)}$ is diameter of tree in centimeter at ground level
$D_{(0.3\ m)}$ is diameter of tree in centimeter at 0.3 m height

For instance, if $D_{(0.3\ m)}$ was 70 cm, the $D_{(ground)}$ is calculated as;

$D_{(ground)}$ = 70 cm + 10% of 70 cm
= 70 + 7
= 77 cm

The most commonly used formulae for calculating volume are the Huber, Newton and Smalian's formulae (Sadiq, 2006, and Goulding, 1979). Of the three commonly used volume calculation approaches or formulae, the Smalian's formula has been used to calculate volume (in m^3) for this study, which is;

$$\text{Section volume } (V_s) = \frac{A+a}{2} * L \quad (3)$$

Where A = Cross sectional area at large end of the section
a = Cross sectional area at small end of the section
L = Length of the section

Smalian's formula is the easiest and least expensive to apply and therefore applied to get volume for each section of the sample trees. However, for the terminal section, the following formula was used to calculate the volume;

$$\text{Terminal section volume } (V_t) = \frac{A}{3} * L \quad (4)$$

The volume for sections and terminal section for individual trees were then summed to obtain the total volume for each individual sample tree, which is;

$$\text{Volume of tree (V)} = \sum_{s=1}^{n} V_s + V_t \qquad (5)$$

After obtaining individual tree volume (Volume.m3), it was then tabulated against the variables - height in meter (Height.m) and the diameter at breast height in centimeter (DBH.cm).

4. The Dataset used for modeling volume of *Pinus wallichiana*

A total 249 trees have been felled and collected data for developing merchantable volume equation for *Pinus wallichiana*. From each regions: eastern, eastern central, western and western central, 8 trees each were felled and collected data as part of field work for biomass equation development. However, in order to improve the model, additional samples were collected from forest management units (FMUs) and outside FMU areas within Paro, Thimphu and Haa Dzongkhags. Summary of the dataset is presented below, while the detailed dataset is provided as an annexure.

4.1 Summary descriptive statistics of *Pinus wallichiana* dataset

```
> summary(pw)
Tree_ID          Height.m             DBH.cm            Volume.m3
pwec01 :  1    Min.    : 4.60      Min.    : 10.0     Min.    : 0.01813
pwec02 :  1    1st Qu.:12.00       1st Qu.: 19.9      1st Qu.: 0.15496
pwec03 :  1    Median :16.00       Median : 27.8      Median : 0.38207
pwec04 :  1    Mean    :18.73      Mean    : 34.9     Mean    : 1.51919
pwec05 :  1    3rd Qu.:24.00       3rd Qu.: 47.6      3rd Qu.: 1.86945
pwec07 :  1    Max.    :61.30      Max.    :128.0     Max.    :16.71401

BA.m2                  BAH.m3                 DBH2H.m3
Min.    :0.0079        Min.    : 0.057        Min.    : 0.073
1st Qu.:0.0311         1st Qu.: 0.375         1st Qu.: 0.478
Median :0.0607         Median : 0.871         Median : 1.108
Mean    :0.1269        Mean    : 3.437        Mean    : 4.376
3rd Qu.:0.1780         3rd Qu.: 4.677         3rd Qu.: 5.955
Max.    :1.2868        Max.    :43.751        Max.    :55.706
```

5. Fitting the models

The models have been fitted in R, which is a robust statistical computing environment. It is a powerful tool which provides wide range of statistical and graphical options to explore, calculate and manage data besides modelling. It is very powerful and widely used statistical tool which is free and allows user to customize the scripts depending on desired output, which is not possible in many of the statistical softwares.

After reading in the excel files into R, we created other variables namely; basal area in square meter (BA.m2), basal area in meter times height in meter (BAH.m3) and square of the diameter in meter times height in meter (DBH2H.m3). The height in meter (Height.m) and diameter in centimeter (DBH.cm) were measured and recorded in the field.

Prior to fitting models, we explored and examined each set of data by preparing descriptive summaries that provided mean, median and range of dependent/response and independent variables. Then we plotted scatter graphs which provided sense of relationship between the dependent/response (volume) and independent/predictor variables (namely DBH.cm, BA.m2, DBH2H.m3 and BAH.m3). These graphs showed curvilinear relationship between response and predictor variables. The scatter plots also clearly revealed the presence of phenomenon, referred in statistical parlance, as heteroscedasticity, which is the increase in variation in response (volume) variable with increase in value of the predictor variables.

Therefore, we fitted the models using the gls ( ) function of the nlme package of R, because the gls ( ) function has the capability to model heteroscedasticity. We didn't transform the variables, mainly response variable, because transformation makes it difficult to directly interpret the relationship between response and predictor variables; and secondly to compare the AIC and BIC values among the different models, the response variables need to be identical.

The models were fitted with volume as a function of four variables;

1) DBH.cm,
2) BA.m2,
3) DBH2H.m3 and
4) BAH.m3.

For each of the variable, we fitted one simple gls ( ) function, which can be written in the following form;

$$Y = \beta_0+\beta_1X + \varepsilon, \quad (6)$$

Where Y = Volume (V) and X = predictor variable

And then fitted 3 models with restricted natural spline functions. The restricted natural cubic spline function enables better tracking of curvilinear relationship between response and predictor variables. These models introduce an additional predictor variable as part of a 3 knot-cubic spline. They take the following form;

$$Y = \beta_0+\beta_1X_1 + \beta_2X_2 + \varepsilon, \quad (7)$$

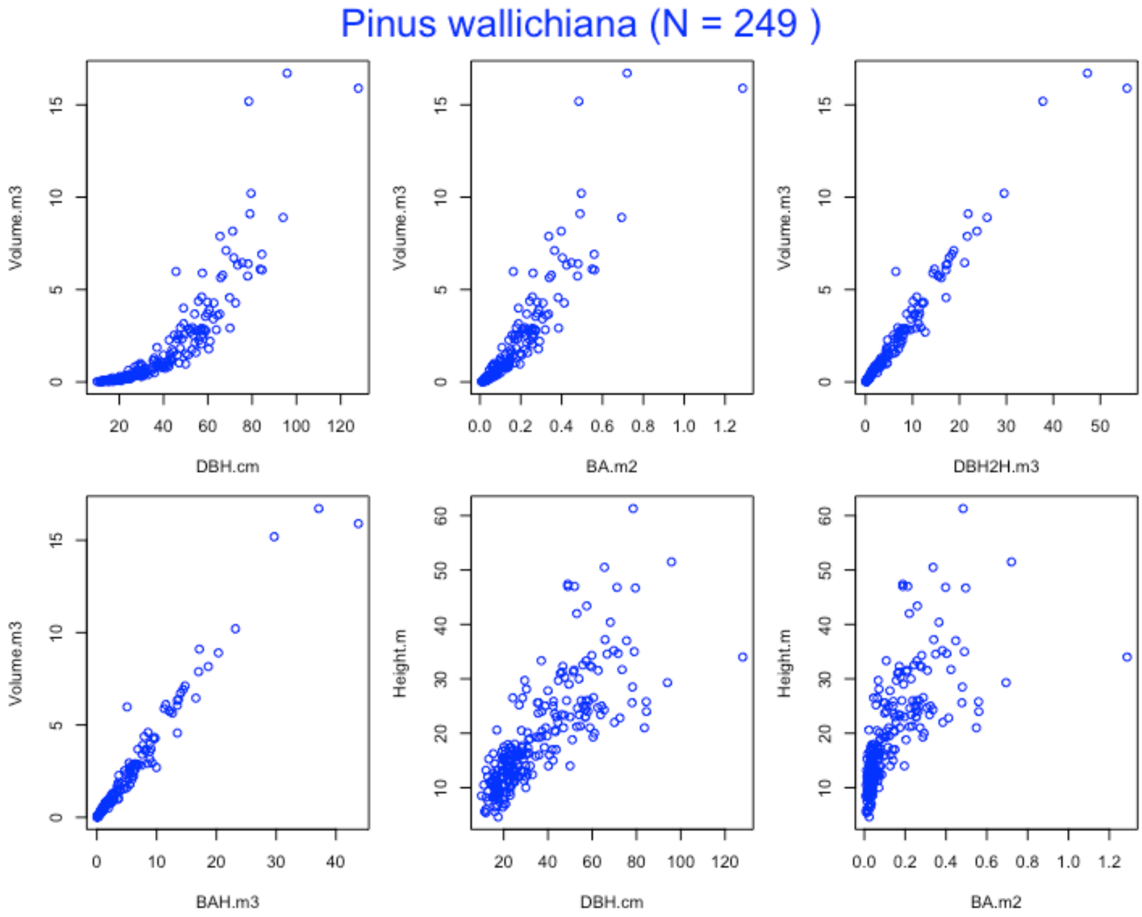
Where Y = Response variable, volume (V)

X_1 = Predictor variable

$X_2 = g(X_1)$

And $g(X_1)$ is the spline transformation of X_1 predictor variable

6. Summary Plots



7. Models and results

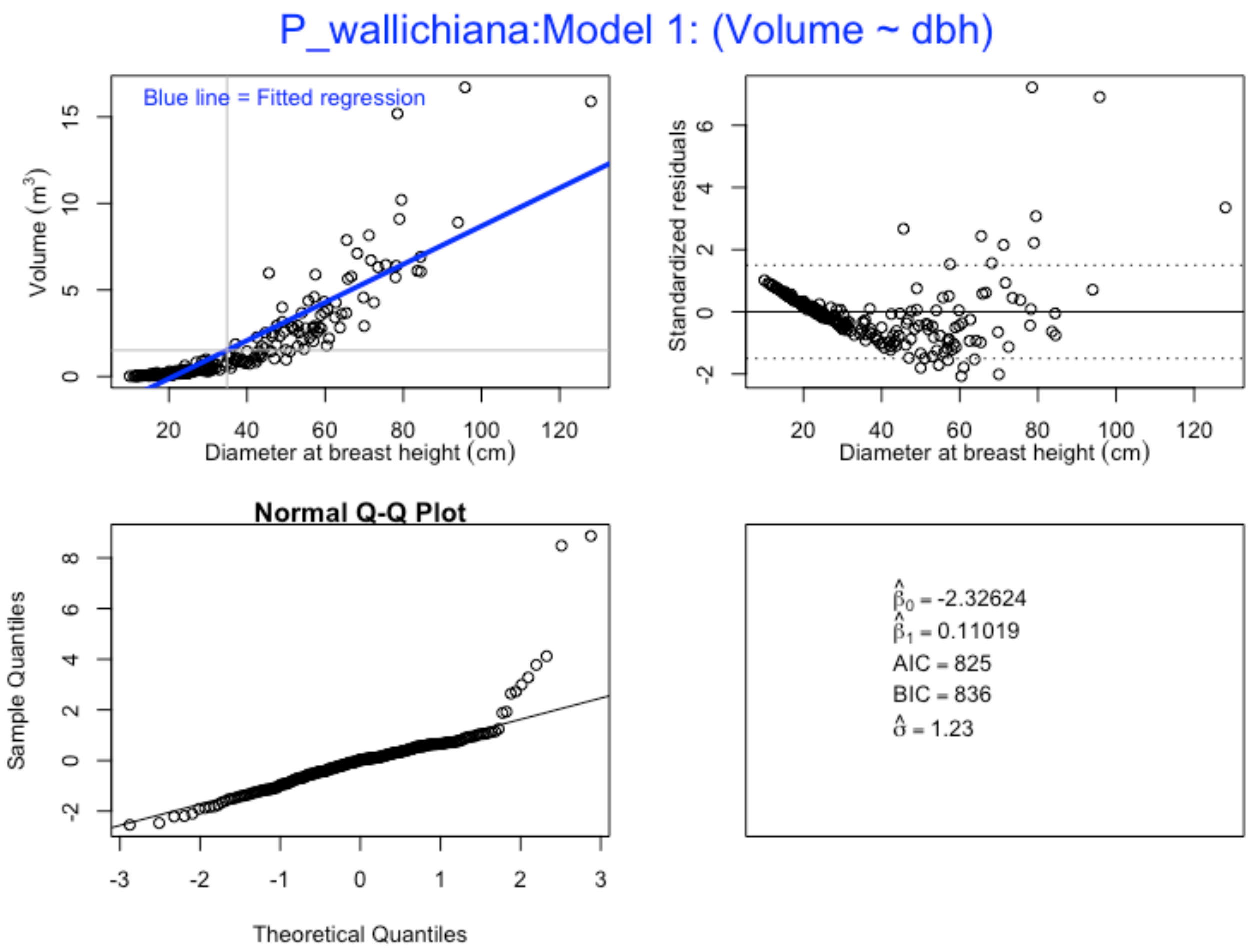
7.1 Model 1 - Volume with diameter at breast height (DBH) as predictor

```
> pw.m1 <- gls(Volume.m3 ~ DBH.cm)
> summary(pw.m1)

Generalized least squares fit by REML
  Model: Volume.m3 ~ DBH.cm
  Data: NULL
       AIC      BIC   logLik
  825.3701 835.8983 -409.685

Coefficients:
                Value  Std.Error   t-value p-value
(Intercept) -2.326239 0.15679850 -14.83585       0
DBH.cm       0.110193 0.00390088  28.24828       0
```

Plot of model 1



7.2 Model 2 - Volume with diameter at breast height (DBH) as predictor, with varFixed

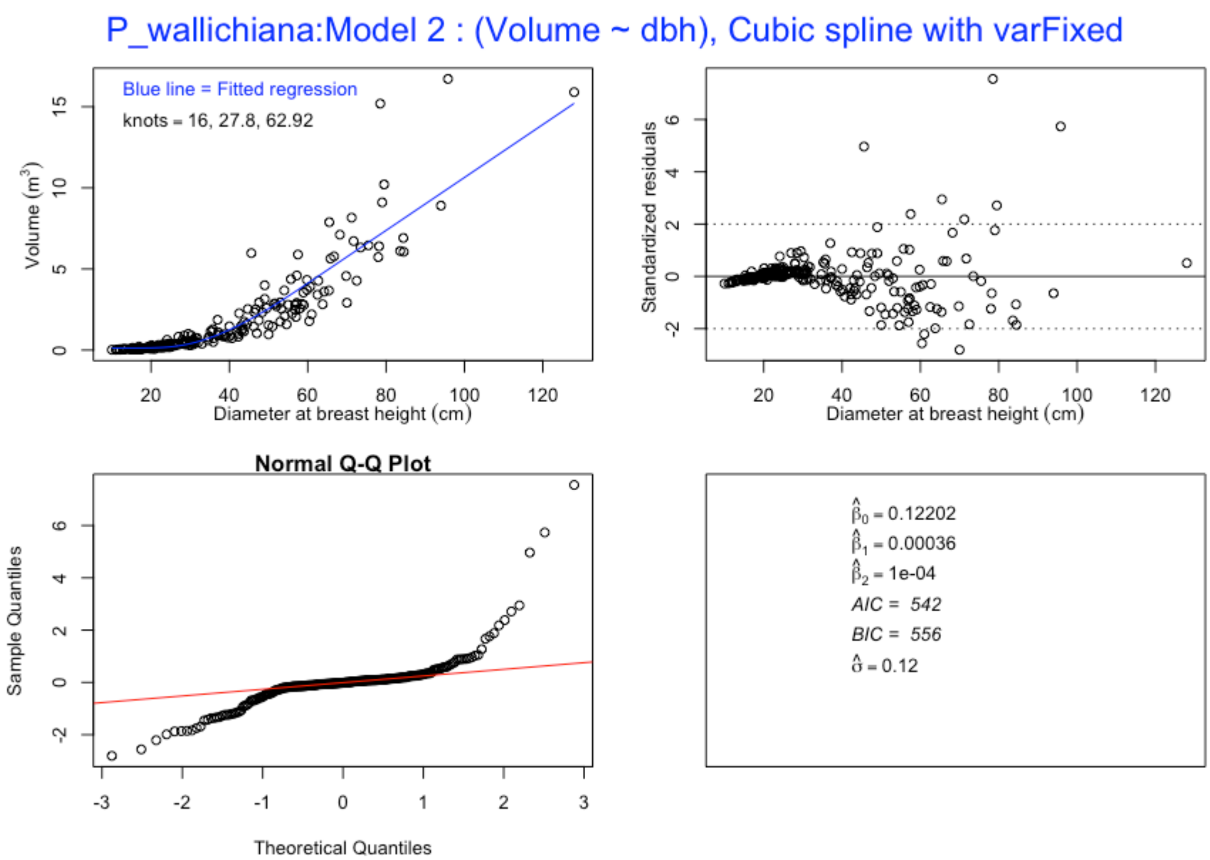
```
> pw.m2 <- gls(Volume.m3 ~ DBH.cm + DBH.cm.splinepoints,
             na.action=na.omit, weights = varFixed(~DBH.cm))
> summary(pw.m2)
```

```
Generalized least squares fit by REML
  Model: Volume.m3 ~ DBH.cm + DBH.cm.splinepoints
  Data: NULL
      AIC      BIC    logLik
  541.895 555.9164 -266.9475

Variance function:
 Structure: fixed weights
 Formula: ~DBH.cm

Coefficients:
                         Value  Std.Error   t-value p-value
(Intercept)         0.12201663 0.15330593  0.795903  0.4269
DBH.cm              0.00035913 0.00716122  0.050149  0.9600
DBH.cm.splinepoints 0.00009795 0.00000717 13.652731  0.0000
```

Plot of Model 2



7.3 Model 3- Volume with diameter at breast height (DBH) as predictor, with varPower

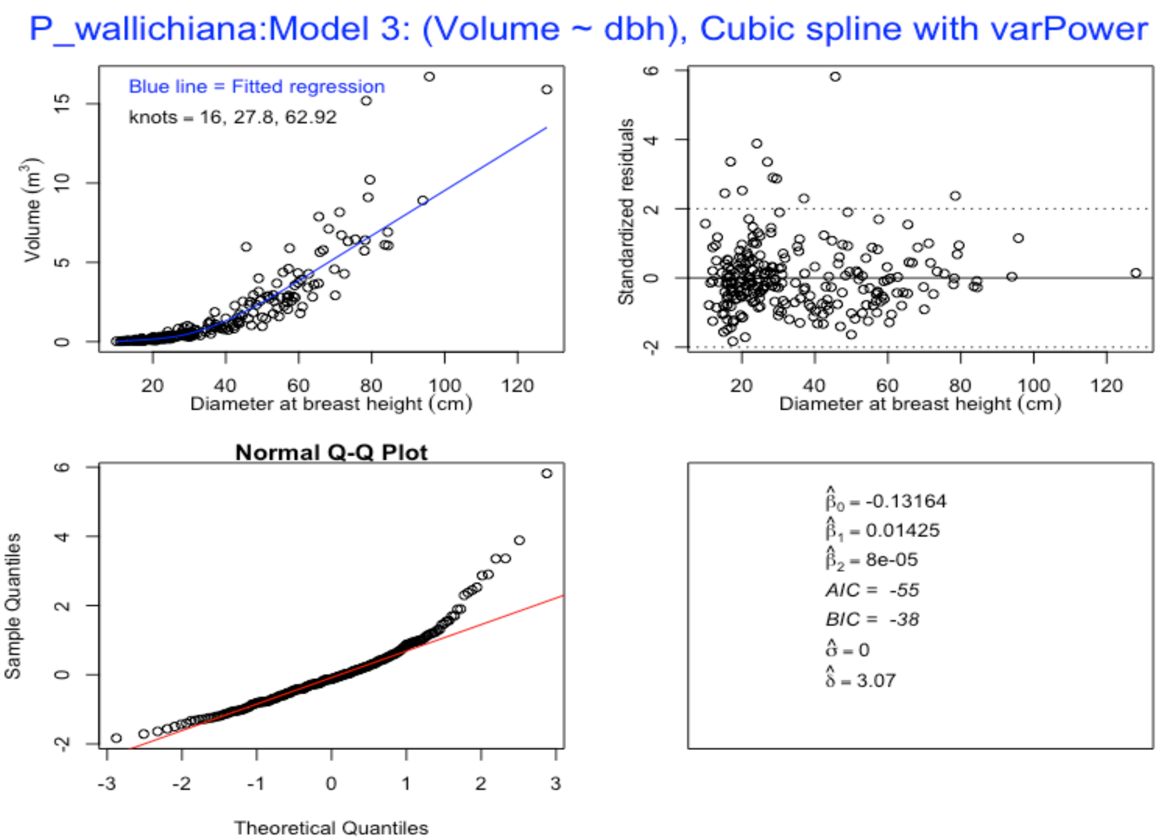
```
> pw.m3 <- gls(Volume.m3 ~ DBH.cm + DBH.cm.splinepoints,
           na.action=na.omit, weights = varPower(form =
           ~DBH.cm))
> summary(pw.m3)

Generalized least squares fit by REML
  Model: Volume.m3 ~ DBH.cm + DBH.cm.splinepoints
  Data: NULL
        AIC       BIC   logLik
  -55.28813 -37.76148 32.64407

Variance function:
 Structure: Power of variance covariate
 Formula: ~DBH.cm
 Parameter estimates:
   power
3.066063

Coefficients:
                          Value   Std.Error   t-value p-value
(Intercept)         -0.13164051 0.010817595 -12.16911       0
DBH.cm               0.01424960 0.000764157  18.64749       0
DBH.cm.splinepoints  0.00007698 0.000004359  17.65953       0
DBH.cm               0.0353727 0.00485745     7.282165      0
DBH.cm.splinepoints  0.0000309 0.00000503    6.149941       0
```

Plot of Model 3



Model 4 - Volume with diameter at breast height (DBH) as predictor, with varConstPower

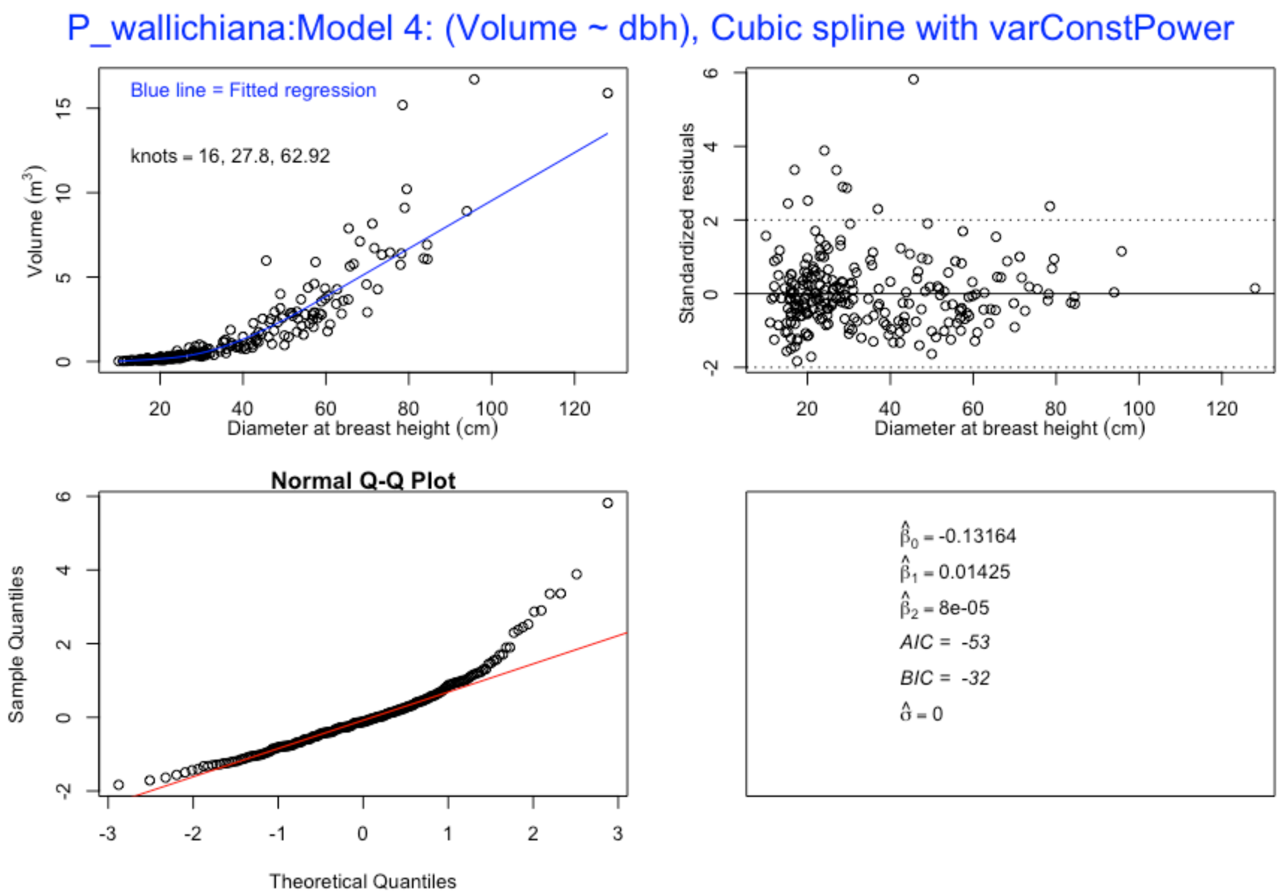
```
> pw.m4 <- gls(Volume.m3 ~ DBH.cm + DBH.cm.splinepoints,
            na.action=na.omit, weights = varConstPower(form =
            ~DBH.cm))
> summary(pw.m4)

Generalized least squares fit by REML
  Model: Volume.m3 ~ DBH.cm + DBH.cm.splinepoints
  Data: NULL
       AIC      BIC    logLik
  82.30686 89.85544 -35.15343

Variance function:
 Structure: Constant plus power of variance covariate
 Formula: ~DBH.cm
 Parameter estimates:
      const       power
5610.541605    2.777108

Coefficients:
                         Value  Std.Error   t-value p-value
(Intercept)         -0.5002530 0.10356140 -4.830497   1e-04
DBH.cm               0.0422143 0.00547699  7.707561   0e+00
DBH.cm.splinepoints  0.0000278 0.00000556  4.998474   0e+00
```

Plot of Model 4



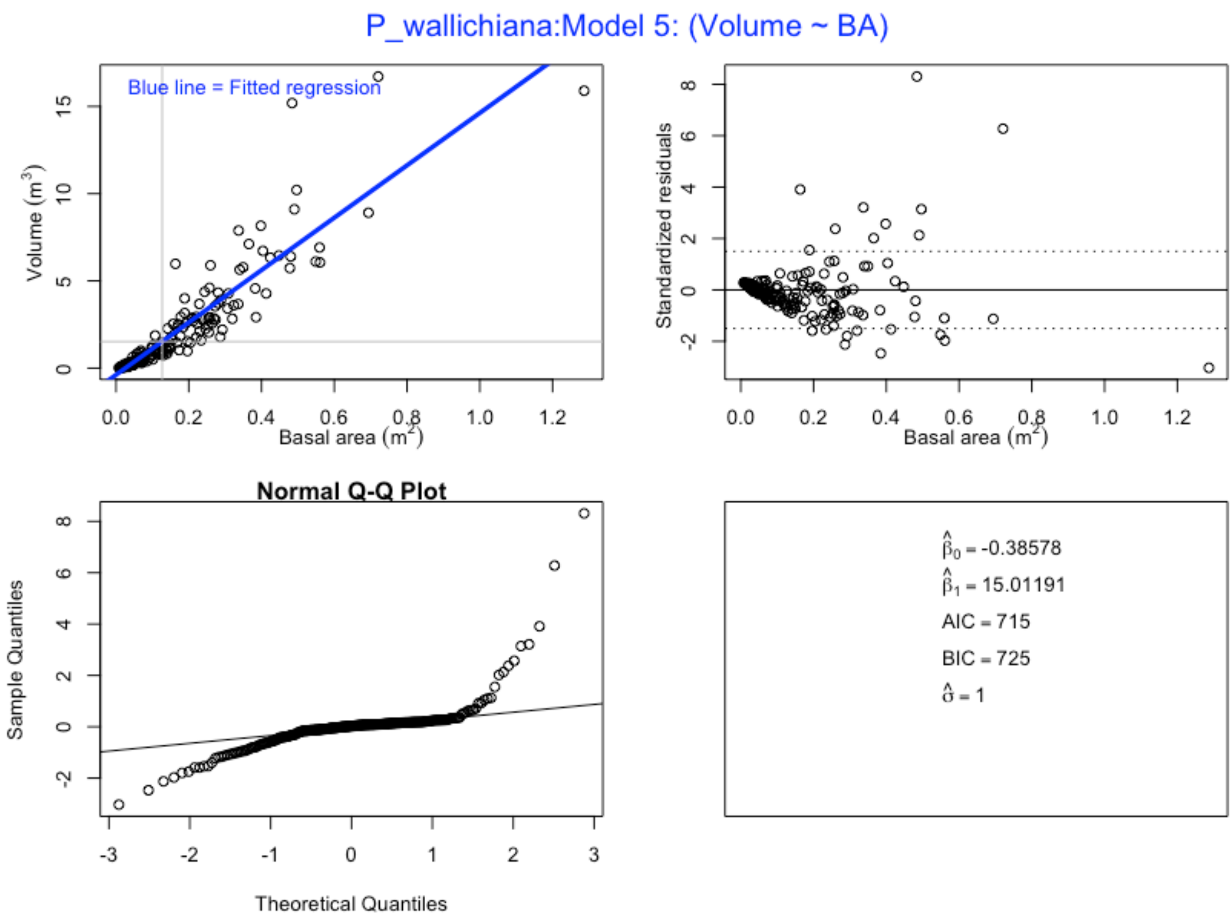
7.5 Model 5 - Volume with basal area (BA) as predictor

```
> pw.m5 <- gls(Volume.m3 ~ BA.m2)
> summary(pw.m5)

Generalized least squares fit by REML
  Model: Volume.m3 ~ BA.m2
  Data: NULL
       AIC      BIC    logLik
  714.7704 725.2985 -354.3852

Coefficients:
                Value Std.Error  t-value p-value
(Intercept) -0.385781 0.0822395 -4.69094       0
BA.m2       15.011910 0.4124054 36.40086       0
```

Plot of Model 5



7.6 Model 6 - Volume with basal area (BA) as predictor, with varFixed

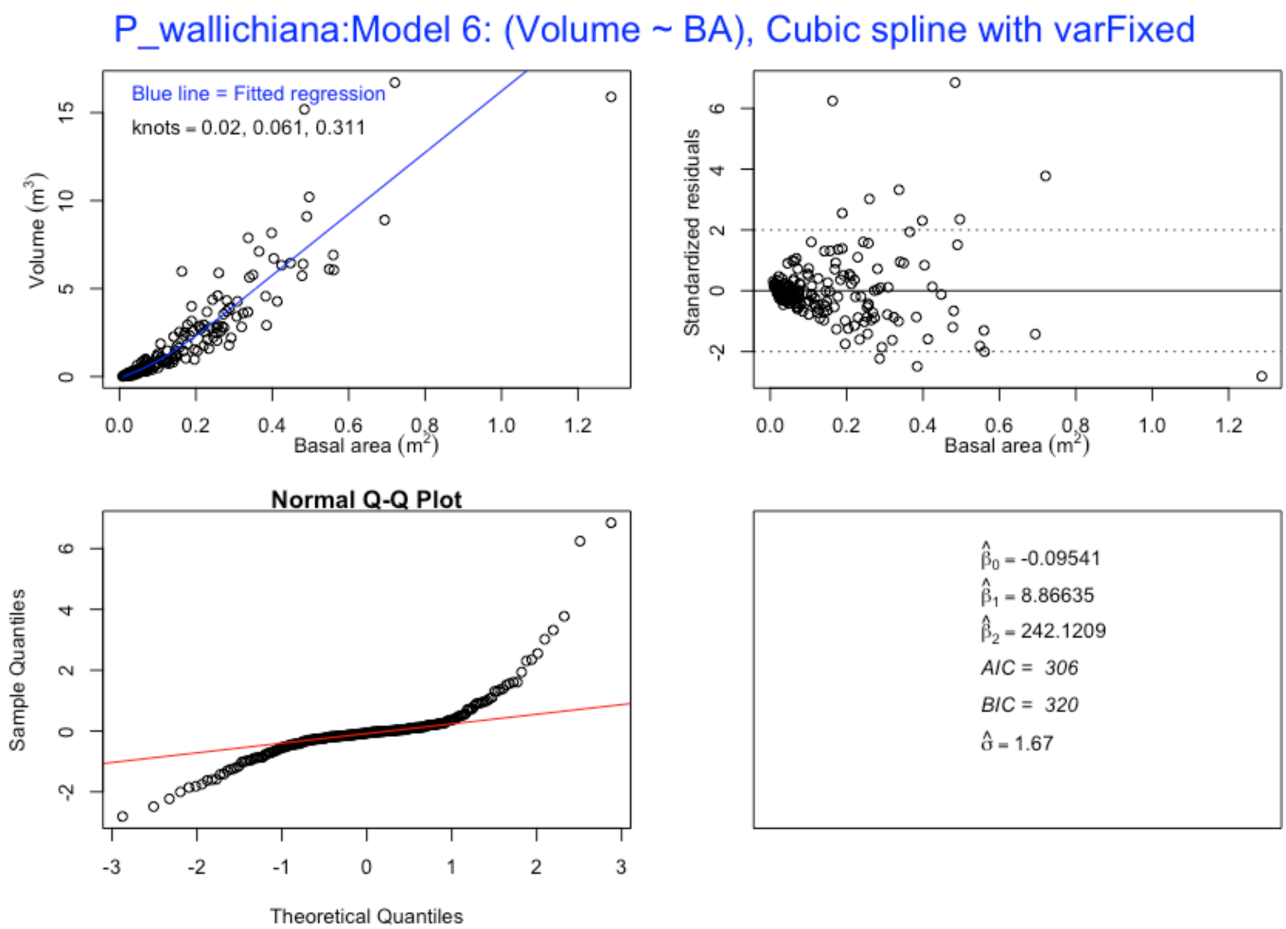
```
> pw.m6<- gls(Volume.m3 ~ BA.m2 + BA.m2.splinepoints,
            na.action=na.omit, weights = varFixed(~BA.m2))
> summary(pw.m6)

Generalized least squares fit by REML
  Model: Volume.m3 ~ BA.m2 + BA.m2.splinepoints
  Data: NULL
      AIC      BIC    logLik
  306.239 320.2603 -149.1195

Variance function:
 Structure: fixed weights
 Formula: ~BA.m2

Coefficients:
                       Value Std.Error   t-value p-value
(Intercept)         -0.09541   0.03927 -2.429619  0.0158
BA.m2                8.86635   0.98874  8.967339  0.0000
BA.m2.splinepoints 242.12091  43.69394  5.541293  0.0000
```

Plot of Model 6



7.7 Model 7 Volume with basal area (BA) as predictor, with varPower

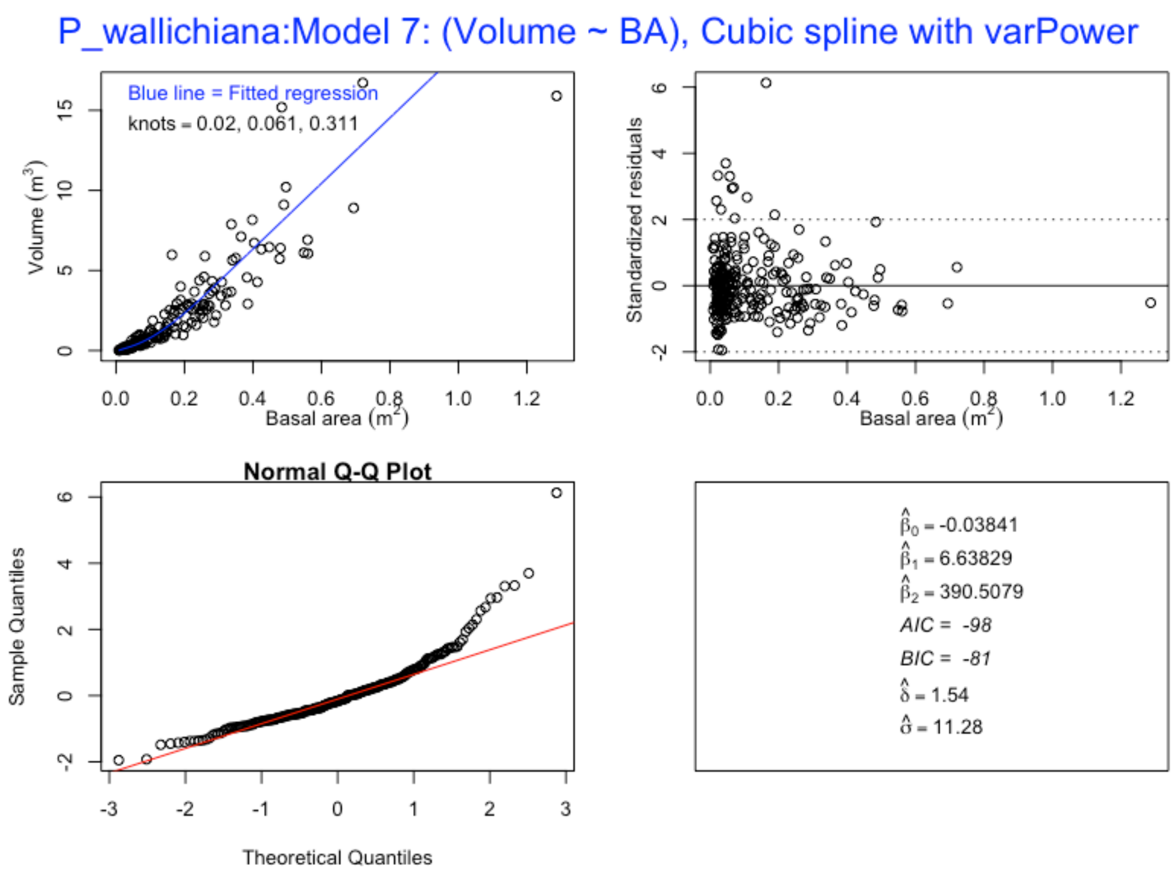
```
> pw.m7 <- gls(Volume.m3 ~ BA.m2 + BA.m2.splinepoints,
              na.action=na.omit, weights = varPower(form = ~BA.m2))
> summary(pw.m7)

Generalized least squares fit by REML
  Model: Volume.m3 ~ BA.m2 + BA.m2.splinepoints
  Data: NULL
        AIC       BIC   logLik
  -98.23399 -80.70733 54.11699

Variance function:
 Structure: Power of variance covariate
 Formula: ~BA.m2
 Parameter estimates:
   power
1.535762

Coefficients:
                      Value Std.Error   t-value p-value
(Intercept)         -0.0384   0.00517 -7.433734       0
BA.m2                6.6383   0.28313 23.445894       0
BA.m2.splinepoints 390.5079  40.61772  9.614223       0
```

Plot of Model 7



7.8 Model 8 – Volume with basal area (BA) as predictor, with varConstPower

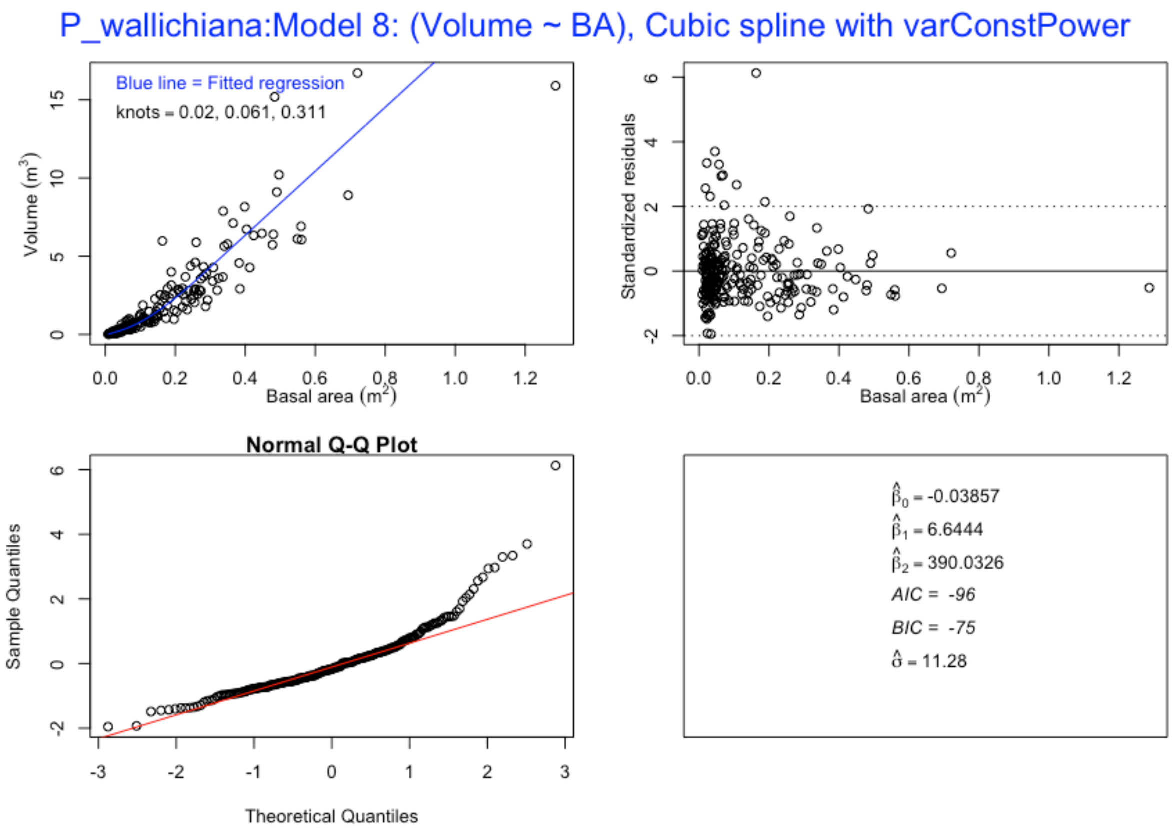
```
> pw.m8 <- gls(Volume.m3 ~ BA.m2 + BA.m2.splinepoints,
               na.action=na.omit, weights = varConstPower(form =
                ~BA.m2))
> summary(pw.m8)
```

```
Generalized least squares fit by REML
  Model: Volume.m3 ~ BA.m2 + BA.m2.splinepoints
  Data: NULL
        AIC       BIC  logLik
  -96.2734 -75.24141 54.1367

Variance function:
 Structure: Constant plus power of variance covariate
 Formula: ~BA.m2
 Parameter estimates:
       const        power
7.253231e-12 1.536039e+00

Coefficients:
                      Value Std.Error   t-value p-value
(Intercept)         -0.0386   0.00518 -7.449876       0
BA.m2                6.6444   0.28346 23.440753       0
BA.m2.splinepoints 390.0326  40.62478  9.600856       0
```

Plot of Model 8



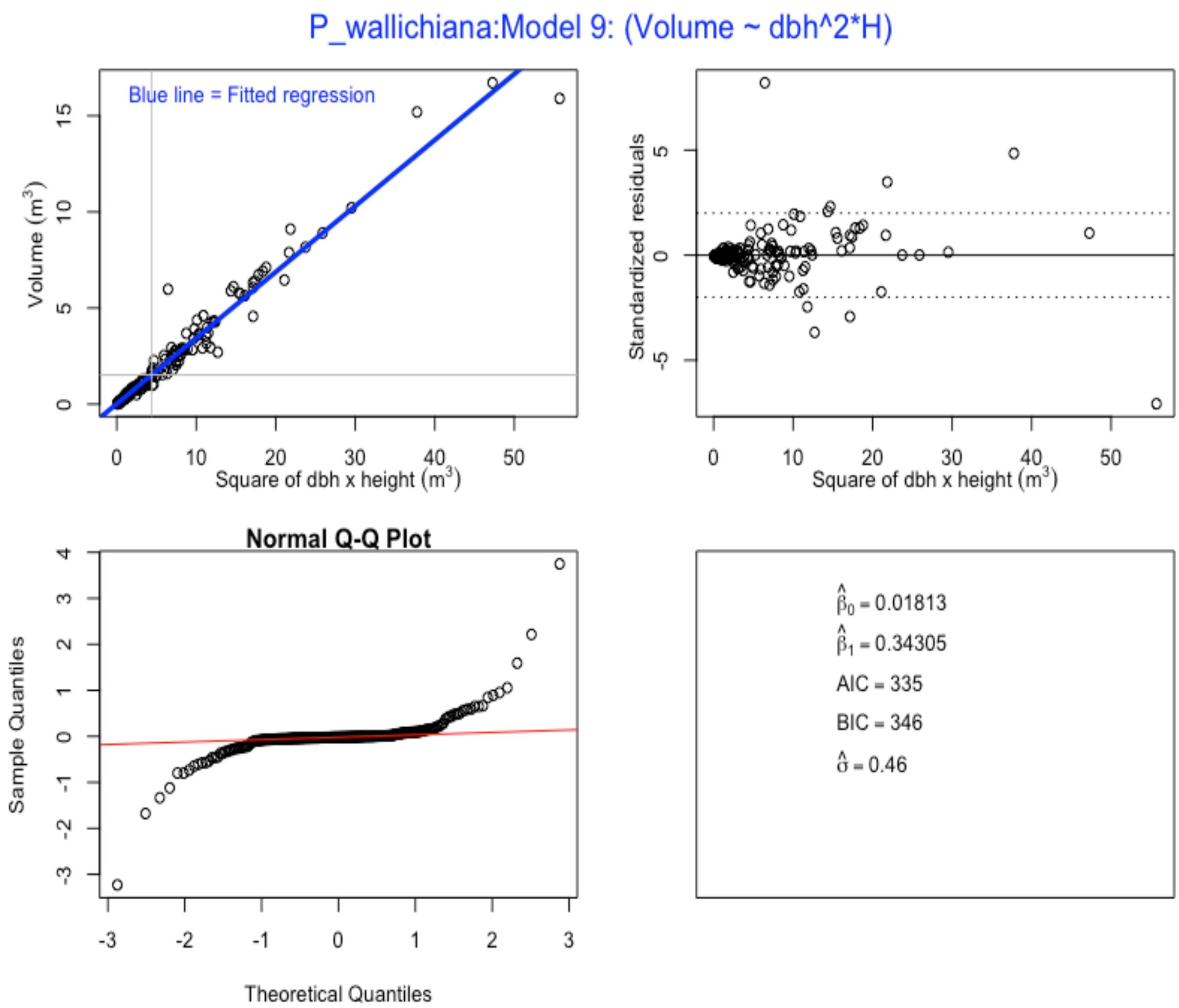
7.9 Model 9 – Volume with square of diameter at breast height * height (DBH2H) as predictor

```
> pw.m9 <- gls(Volume.m3 ~ DBH2H.m3)
> summary(pw.m9)

Generalized least squares fit by REML
  Model: Volume.m3 ~ DBH2H.m3
  Data: NULL
       AIC       BIC     logLik
  335.0587 345.5868 -164.5293

Coefficients:
                Value  Std.Error   t-value p-value
(Intercept) 0.0181276 0.03387269  0.53517   0.593
DBH2H.m3    0.3430474 0.00401577 85.42509   0.000
```

Plot of Model 9



7.10 Model 10– Volume with square of diameter at breast height * height (DBH2H) as predictor, with varFixed

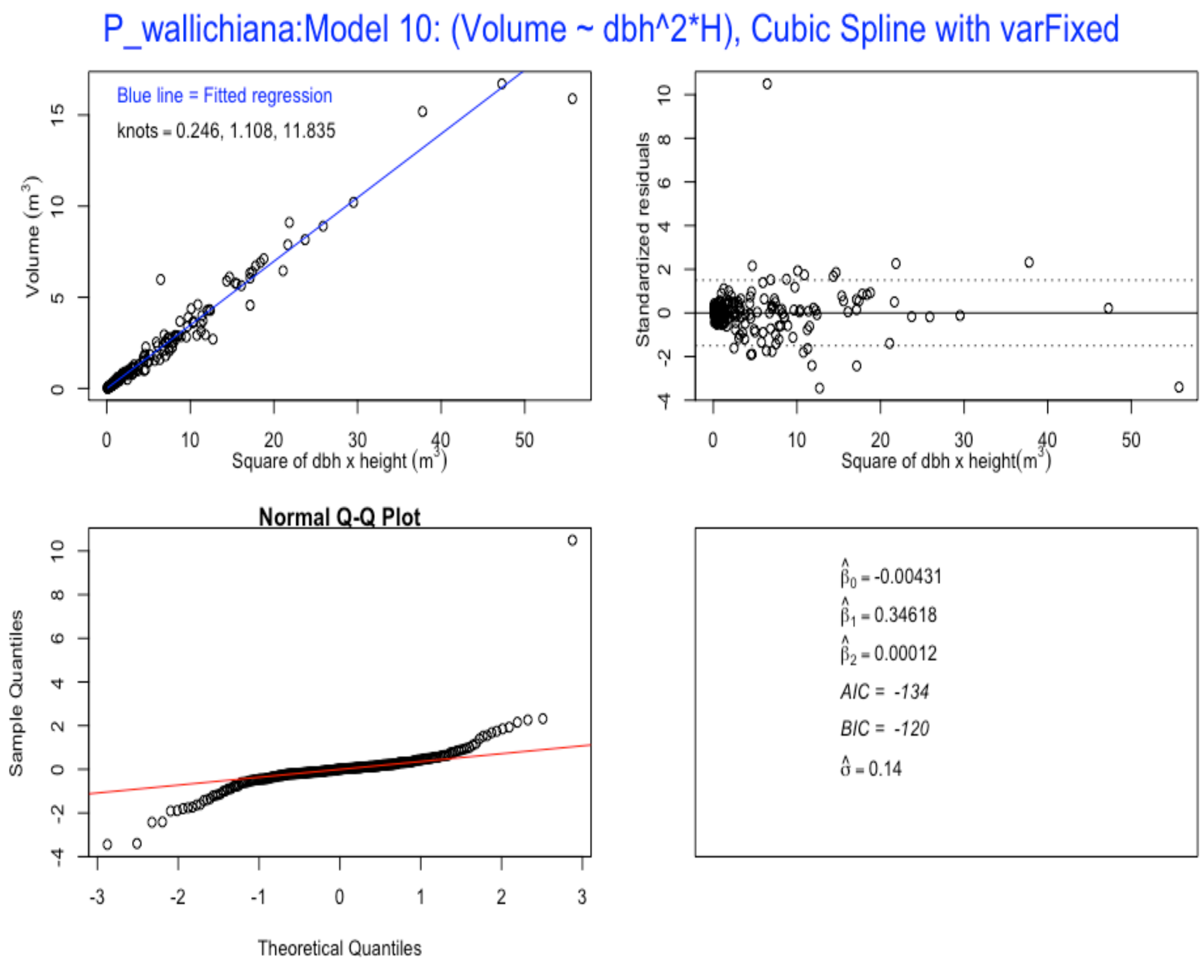
```
> pw.m10 <- gls(Volume.m3 ~ DBH2H.m3 + DBH2H.m3.splinepoints,
            na.action=na.omit, weights = varFixed(~DBH2H.m3))
> summary(pw.m10)

Generalized least squares fit by REML
  Model: Volume.m3 ~ DBH2H.m3 + DBH2H.m3.splinepoints
  Data: NULL
        AIC       BIC   logLik
  -134.4981 -120.4768 71.24905

Variance function:
 Structure: fixed weights
 Formula: ~DBH2H.m3

Coefficients:
                           Value   Std.Error   t-value p-value
(Intercept)           -0.0043088 0.009058774 -0.475653  0.6347
DBH2H.m3               0.3461832 0.012672173 27.318376  0.0000
DBH2H.m3.splinepoints  0.0001172 0.000626634  0.187013  0.8518
```

Plot of Model 10



7.11 Model 11– Volume with square of diameter at breast height * height (DBH2H) as predictor, with varPower

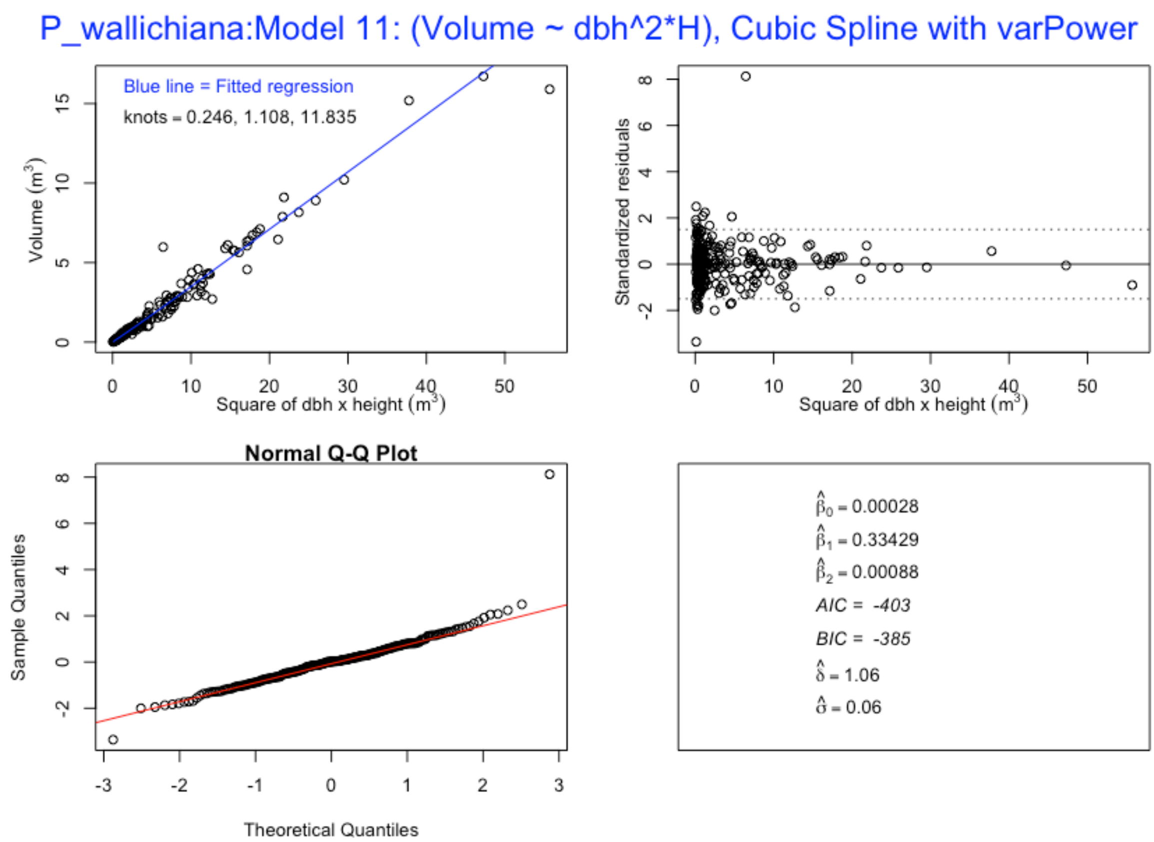
```
> pw.m11 <- gls(Volume.m3 ~ DBH2H.m3 + DBH2H.m3.splinepoints,
              na.action=na.omit, weights = varPower(form =
              ~DBH2H.m3))
> summary(pw.m11)

Generalized least squares fit by REML
  Model: Volume.m3 ~ DBH2H.m3 + DBH2H.m3.splinepoints
  Data: NULL
        AIC       BIC   logLik
  -402.9307 -385.4041 206.4654

Variance function:
 Structure: Power of variance covariate
 Formula: ~DBH2H.m3
 Parameter estimates:
   power
1.055553

Coefficients:
                          Value   Std.Error  t-value p-value
(Intercept)           0.0002808 0.001976857  0.14203  0.8872
DBH2H.m3              0.3342877 0.007122339 46.93510  0.0000
DBH2H.m3.splinepoints 0.0008796 0.000656152  1.34057  0.1813
```

Plot of Model 11



7.12 Model 12 –Volume with square of diameter at breast height * height (DBH2H) as predictor, with varConstPower

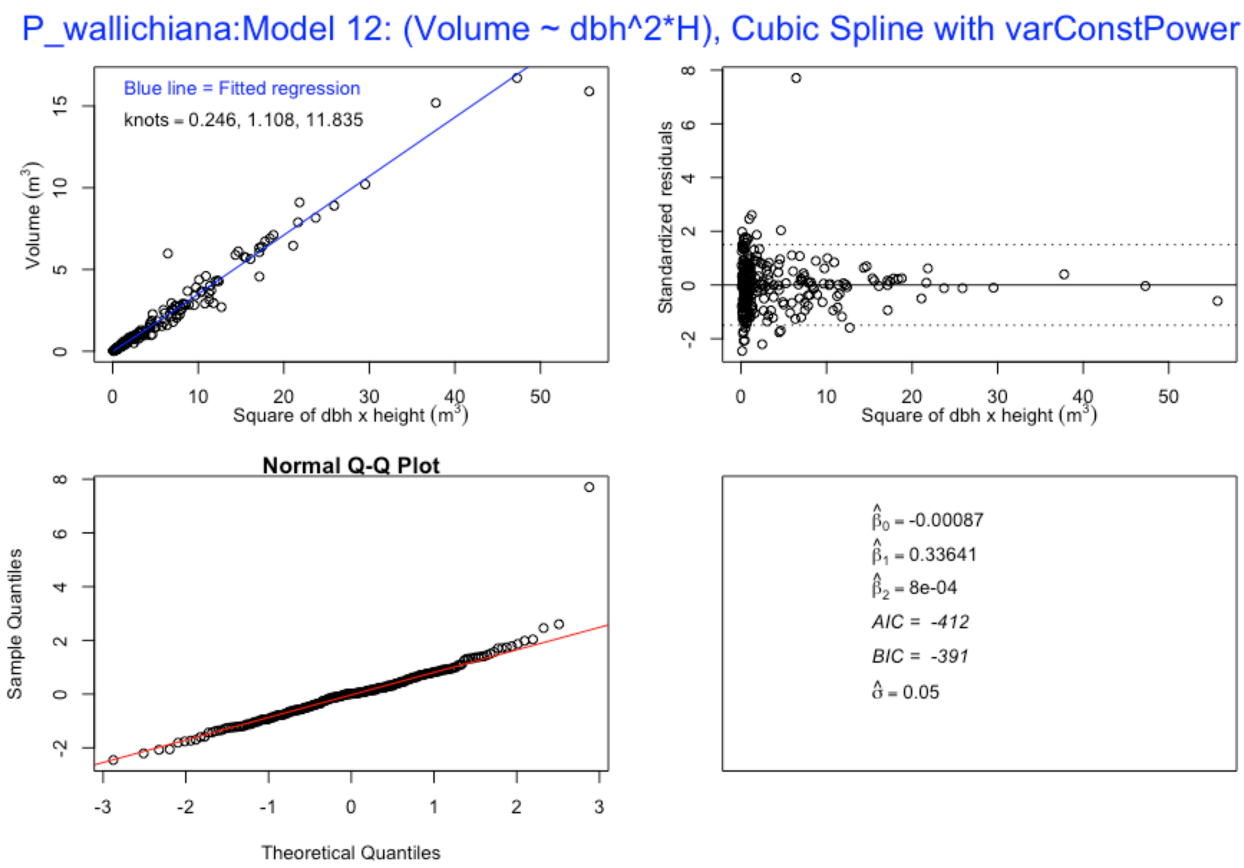
```
> pw.m12 <- gls(Volume.m3 ~ DBH2H.m3 + DBH2H.m3.splinepoints,
               na.action=na.omit, weights = varConstPower(form =
               ~DBH2H.m3))
> summary(pw.m12)

Generalized least squares fit by REML
  Model: Volume.m3 ~ DBH2H.m3 + DBH2H.m3.splinepoints
  Data: NULL
        AIC       BIC   logLik
  -411.7222 -390.6902 211.8611

Variance function:
 Structure: Constant plus power of variance covariate
 Formula: ~DBH2H.m3
 Parameter estimates:
   const    power
0.119535 1.229481

Coefficients:
                           Value   Std.Error  t-value p-value
(Intercept)           -0.0008658 0.002536199 -0.34137  0.7331
DBH2H.m3               0.3364145 0.007094375 47.41989  0.0000
DBH2H.m3.splinepoints  0.0008009 0.000747453  1.07150  0.2850
```

Plot of Model 12



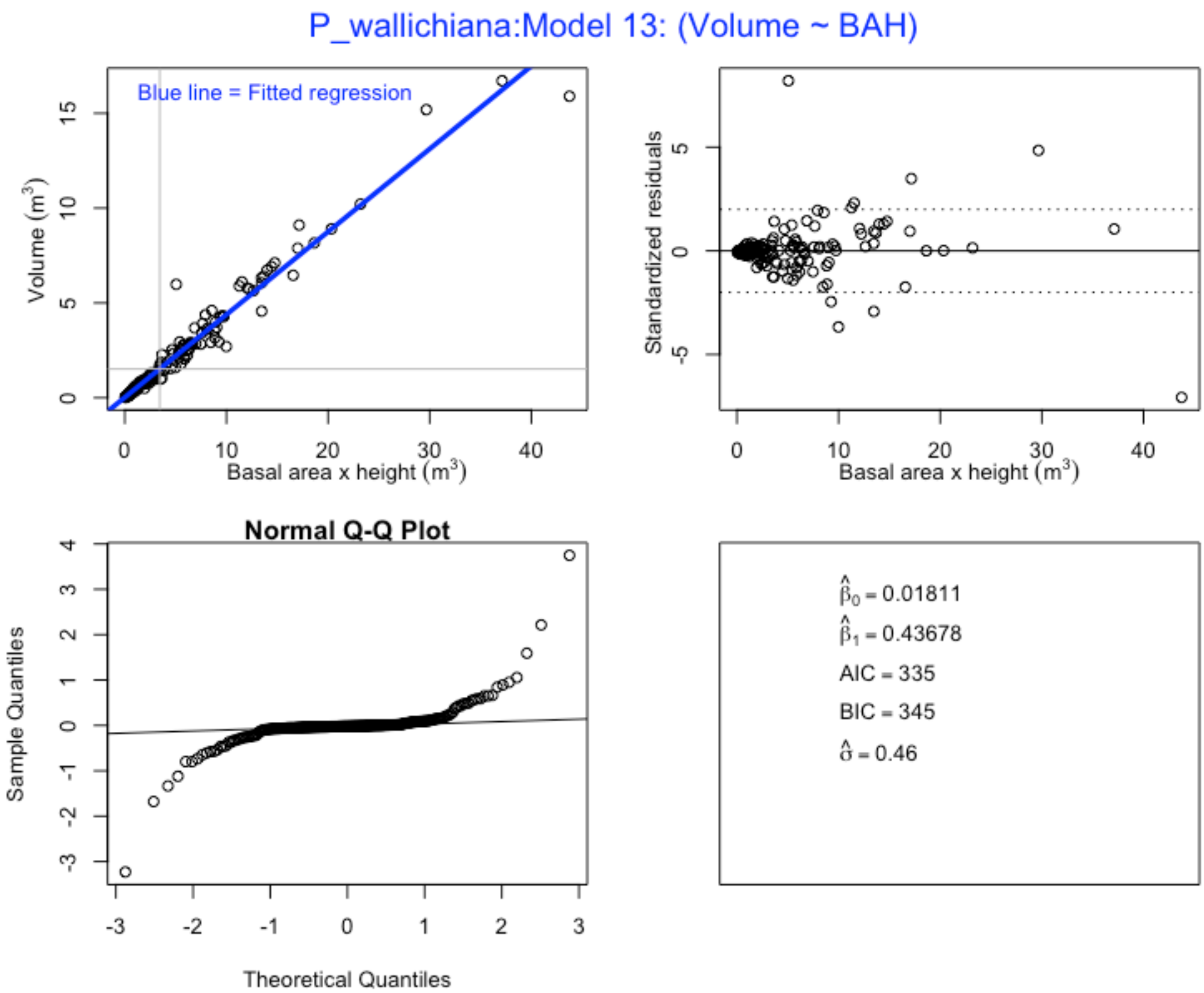
7.13 Model 13 – Volume with basal area * height (BAH) as predictor

```
> pw.m13 <- gls(Volume.m3 ~ BAH.m3)
> summary(pw.m13)

Generalized least squares fit by REML
  Model: Volume.m3 ~ BAH.m3
  Data: NULL
       AIC      BIC    logLik
  334.5427 345.0709 -164.2714

Coefficients:
                Value  Std.Error  t-value p-value
(Intercept) 0.0181119 0.03387052  0.53474  0.5933
BAH.m3      0.4367786 0.00511265 85.43097  0.0000
```

Plot of Model 13



7.14 Model 14 – Volume with basal area * height (BAH) as predictor, with varFixed

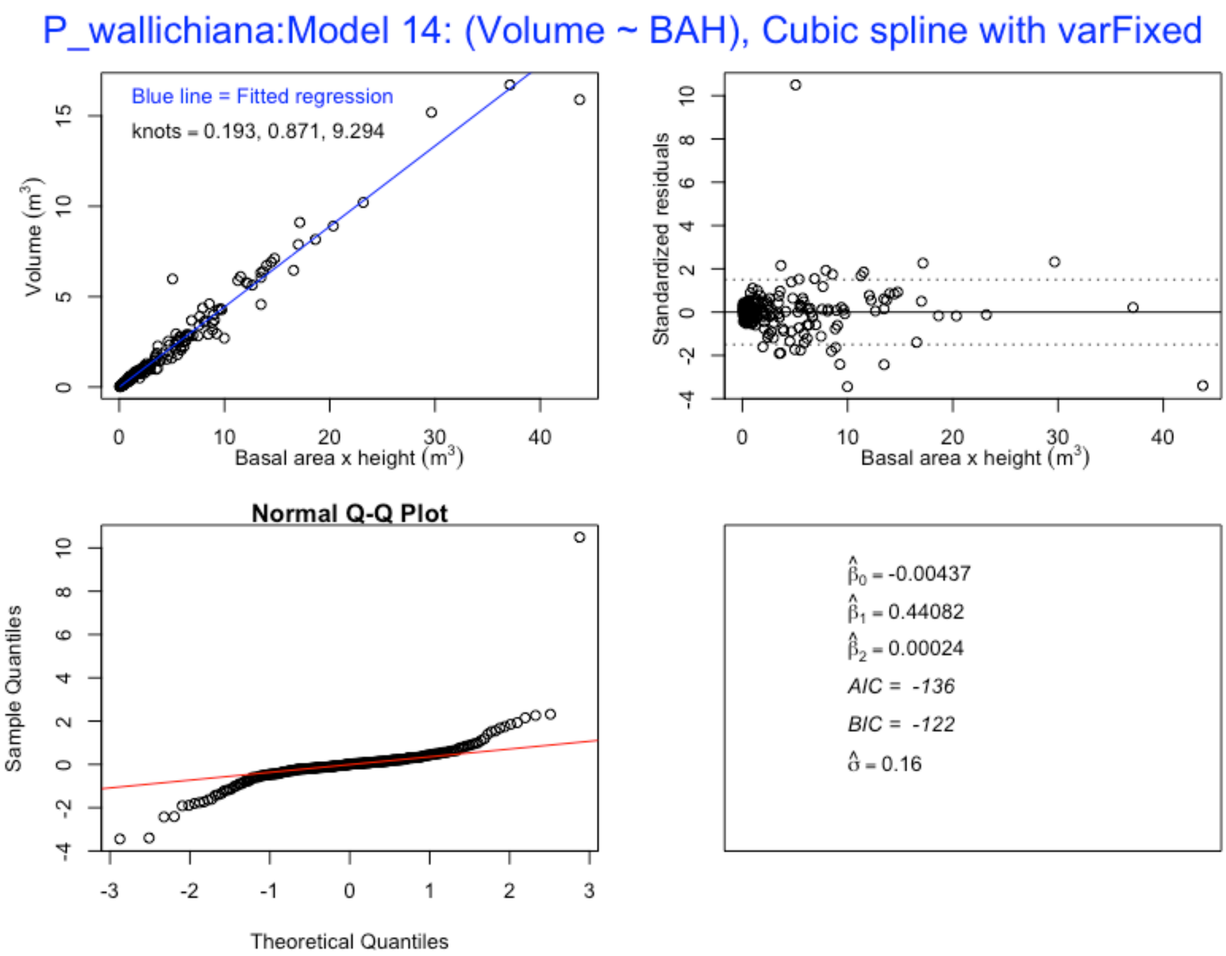
```
> pw.m14 <- gls(Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints,
              na.action=na.omit, weights = varFixed(~BAH.m3))
> summary(pw.m14)

Generalized least squares fit by REML
  Model: Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints
  Data: NULL
        AIC       BIC   logLik
  -136.4189 -122.3976 72.20944

Variance function:
 Structure: fixed weights
 Formula: ~BAH.m3

Coefficients:
                         Value   Std.Error   t-value p-value
(Intercept)         -0.0043652 0.009062309 -0.481684  0.6305
BAH.m3               0.4408229 0.016135818 27.319528  0.0000
BAH.m3.splinepoints  0.0002377 0.001291674  0.184063  0.8541
```

Plot of Model 14



7.15 Model 15– Volume with basal area * height (BAH) as predictor, with varPower

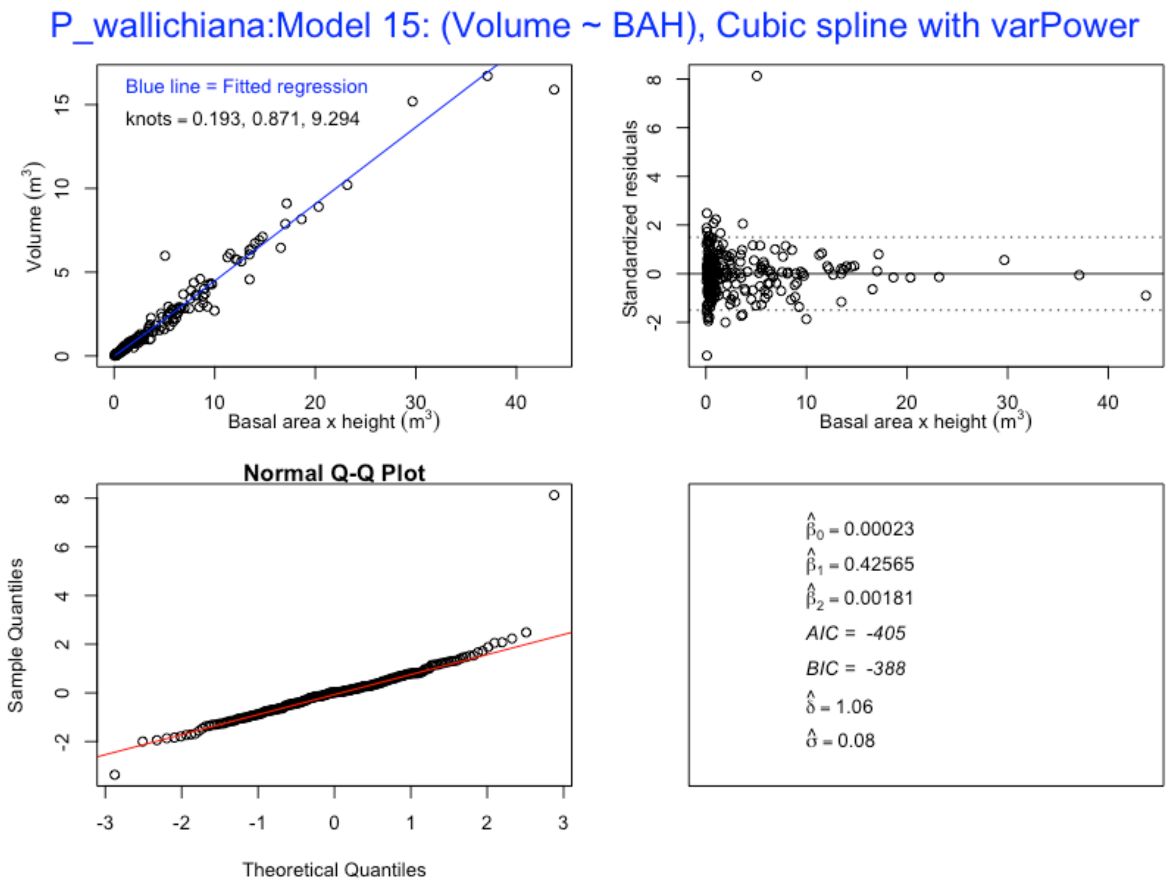
```
> pw.m15 <- gls(Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints,
             na.action=na.omit, weights = varPower(form =
             ~BAH.m3))
> summary(pw.m15)

Generalized least squares fit by REML
  Model: Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints
  Data: NULL
        AIC      BIC   logLik
  -405.1697 -387.643 207.5848

Variance function:
 Structure: Power of variance covariate
 Formula: ~BAH.m3
 Parameter estimates:
   power
1.056248

Coefficients:
                        Value   Std.Error  t-value p-value
(Intercept)         0.0002340 0.001973719  0.11858  0.9057
BAH.m3              0.4256489 0.009056731 46.99807  0.0000
BAH.m3.splinepoints 0.0018121 0.001352515  1.33979  0.1816
```

Plot of Model 15



7.16 Model 16 – Volume with basal area * height (BAH) as predictor, with varConstPower

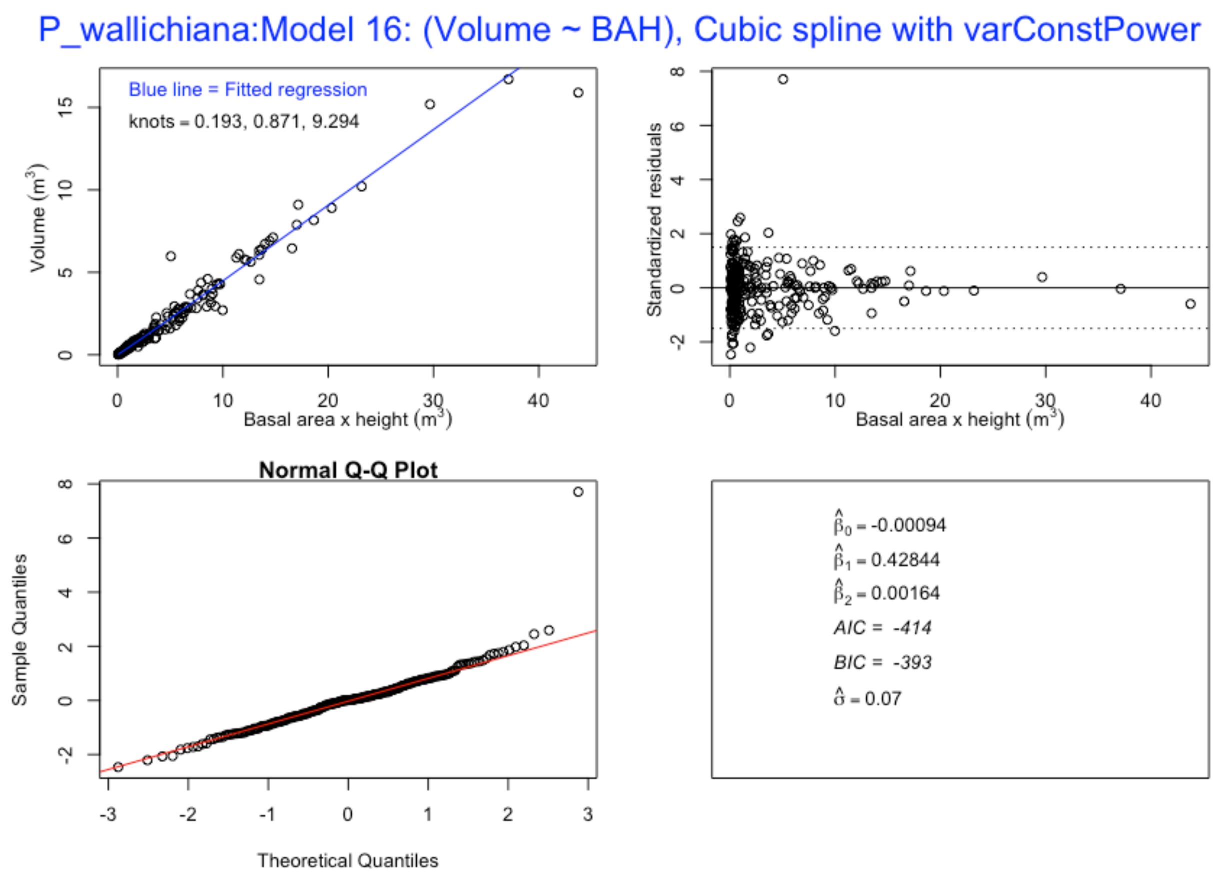
```
> pw.m16 <- gls(Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints,
              na.action=na.omit,weights=varConstPower(form=~BAH.m3)
              )
> summary(pw.m16)

Generalized least squares fit by REML
  Model: Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints
  Data: NULL
        AIC       BIC   logLik
  -413.8571 -392.8251 212.9286

Variance function:
 Structure: Constant plus power of variance covariate
 Formula: ~BAH.m3
 Parameter estimates:
     const      power
0.08839353 1.22926147

Coefficients:
                         Value   Std.Error  t-value p-value
(Intercept)         -0.0009352 0.002534045 -0.36906  0.7124
BAH.m3               0.4284381 0.009029416 47.44915  0.0000
BAH.m3.splinepoints  0.0016409 0.001540180  1.06538  0.2877
```

Plot of Model 16



8. Model evaluation using AIC and BIC values

SN	Model	AIC	BIC
1	Model 1 `> pw.m1 <- gls(Volume.m3 ~ DBH.cm)`	825	835
2	Model 2 `> pw.m2 <- gls(Volume.m3 ~ DBH.cm + DBH.cm.splinepoints, na.action=na.omit, weights = varFixed(~DBH.cm))`	541	555
3	Model 3 `> pw.m3 <- gls(Volume.m3 ~ DBH.cm + DBH.cm.splinepoints, na.action=na.omit, weights = varPower(form = ~DBH.cm))`	-55	-37
4	Model 4 `> pw.m4 <- gls(Volume.m3 ~ DBH.cm + DBH.cm.splinepoints, na.action=na.omit, weights = varConstPower(form = ~DBH.cm))`	82	89
5	Model 5 `> pw.m5 <- gls(Volume.m3 ~ BA.m2)`	714	725
6	Model 6 `> pw.m6<- gls(Volume.m3 ~ BA.m2 + BA.m2.splinepoints, na.action=na.omit, weights = varFixed(~BA.m2))`	306	320
7	Model 7 `> pw.m7 <- gls(Volume.m3 ~ BA.m2 + BA.m2.splinepoints, na.action=na.omit, weights = varPower(form = ~BA.m2))`	-98	-81
8	Model 8 `> pw.m8 <- gls(Volume.m3 ~ BA.m2 + BA.m2.splinepoints, na.action=na.omit, weights = varConstPower(form = ~BA.m2))`	-96	-75
9	Model 9 `> pw.m9 <- gls(Volume.m3 ~ DBH2H.m3)`	335	345
10	Model 10	-134	-120

	`> pw.m10 <-gls(Volume.m3 ~ DBH2H.m3 + DBH2H.m3.splinepoints, na.action=na.omit, weights = varFixed(~DBH2H.m3))`		
11	Model 11 `> pw.m11 <-gls(Volume.m3 ~ DBH2H.m3 + DBH2H.m3.splinepoints, na.action=na.omit, weights = varPower(form = ~DBH2H.m3))`	-402	-385
12	Model 12 `> pw.m12 <- gls(Volume.m3 ~ DBH2H.m3 + DBH2H.m3.splinepoints, na.action=na.omit, weights = varConstPower(form = ~DBH2H.m3))`	-411	-390
13	Model 13 `> pw.m13 <- gls(Volume.m3 ~ BAH.m3)`	334	345
14	Model 14 `> pw.m14 <- gls(Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints, na.action=na.omit, weights = varFixed(~BAH.m3))`	-136	-122
15	Model 15 `> pw.m15 <- gls(Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints, na.action=na.omit, weights = varPower(form = ~BAH.m3))`	-405	-387
16	Model 16 `> pw.m16 <- gls(Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints, na.action=na.omit, weights = varConstPower(form = ~BAH.m3))`	-413	-392

9. Selected Models

The best fitting models have been selected based on Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) values of the fitted models. The BIC value was mainly relied upon as it imposes a stronger penalty for the number of parameters in the model that need to be estimated. Smaller the values of AIC and BIC, better the fit of the model. Therefore, for *Pinus wallichiana*, the selected models are;

1. Model 7 (Model which doesn't use height)
```
pw.m7 <- gls(Volume.m3 ~ BA.m2 + BA.m2.splinepoints,
             na.action=na.omit, weights =   varPower(form = ~BA.m2))
```

2. Model 16 (Model which uses the height)
```
pw.m16 <- gls(Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints,
              na.action=na.omit, weights = varConstPower(form = ~BAH.m3))
```

Two models have been selected for *Pinus wallichiana*, one without height (X_1= BA which is model 7) and one with the height (X_1 = BAH, which is Model 16) as predictor or explanatory variable. Both the models have been fitted with natural (restricted) cubic spline function within a linear model framework. Although, nonlinear models are more flexible, they are more complicated than the linear models. The complications involved and amount of time and efforts spent on fitting nonlinear models often fail to justify by the improvements in the models. Moreover, the models fitted with natural (restricted) cubic spline functions perform well and track the curvilinearity better than nonlinear functions that were examined.

10. Demonstration of use of the selected best fit model

In general, the natural spline predictor with knots represented by t1, t2 and t3 takes the following form;

$$Y = \beta_0 + \beta_1 X + \beta_2 X_s + \varepsilon \tag{8}$$

Where X_S corresponds to value in X as follows:

$$\text{Xs} = \text{g(X)} = (X - t1)_+^3 - (X - t2)_+^3 \frac{(t3-t1)}{(t3-t2)} + (X - t3)_+^3 \frac{(t2-t1)}{(t3-t2)} \tag{9}$$

and the value of the positive part functions depend on the values of the knots as follows;

$$(X - t1)_+^3 = (X - t1)_+^3, \text{ if X > t1 and } (X - t1)_+^3 = 0, \text{ if X<t1} \tag{10}$$

$$(X - t2)_+^3 = (X - t2)_+^3, \text{ if X> t2, and } (X - t2)_+^3 = 0, \text{ if X < t2} \tag{11}$$

$$(X - t3)_+^3 = (X - t3)_+^3, \text{ if X > t3, and } (X - t3)_+^3 = 0, \text{ if X<t3} \tag{12}$$

Where t1, t2 and t3 for the above models are 10^{th}, 50^{th} and 90^{th} percentiles and are called knots. The values of knots differ from species and models.

To demonstrate use of the selected models for *Pinus wallichiana* – model 7, the knots t1, t2 and t3 are 0.02, 0.061 and 0.311 as generated by the model. The model 7 has been fitted with volume as function of basal area in meter square (BA) i.e

$$BA = \pi r^2 \tag{13}$$

where in

$$r^2 = \left[\frac{dbh}{2*100}\right]^2 \tag{14}$$

Where r is radius in meters and dbh is diameter at breast height in centimeters.

Therefore, *Pinus wallichiana* with diameter of 54.1 cm resulting in basal area of 0.22987112 m^2, the volume can be estimated using the above equation (model 7) as below. But first the value of BA.m2 has to be calculated, which is;

$$\text{BA} \quad = \pi r^2 = \frac{\pi * 51.8^2}{200^2} = 0.22987112 \text{ m}^2$$

$$\text{g(X)} \quad = (X - t1)_+^3 - (X - t2)_+^3 \frac{(t3-t1)}{(t3-t2)} + (X - t3)_+^3 \frac{(t2-t1)}{(t3-t2)}$$

$$\text{g(BA)} = (BA - t1)_+^3 - (BA - t2)_+^3 \frac{(t3-t1)}{(t3-t2)} + (BA - t3)_+^3 \frac{(t2-t1)}{(t3-t2)}$$

$$\text{g(BA)} = (0.22987112 - 0.02)_+^3 - (0.22987112 - 0.061)_+^3 \frac{(0.311-0.02)}{(0.311-0.061)} + 0,$$

$$= (0.20987112)_+^3 - (0.16887112)_+^3 \frac{(0.291)}{(0.25)} + 0$$

$$= 0.00924396 – 0.004815775 * 1.164$$

$$= 0.00924396 – 0.00560556$$

$$= 0.0036384$$

Hence, the volume predicted for this tree by the selected model (model 7) is

$$V = \beta_0 + \beta_1 . BA + \beta_2 BA . m_2 + \varepsilon$$

$$= -0.0384 + 6.6383 * 0.22987112 + 390.5079 * 0.0036384$$

$$= -0.0384 + 1.52595 + 1.42082$$

$$= \mathbf{2.908\ m^3}$$

Similarly, to demonstrate model 16 with t1, t2 and t3 of 0.193, 0.871, and 9.294 respectively, we considered this same tree but with height, i.e dbh = 54.1 cm resulting in BA = 0.22987112 m^2 and height (H) = 23.4 m.

$$\text{BAH} = 0.22987112 \text{ x } 23.4$$

$$= 5.37898$$

$$g(X) = (X - t1)^3_+ - (X - t2)^3_+ \frac{(t3-t1)}{(t3-t2)} + (X - t3)^3_+ \frac{(t2-t1)}{(t3-t2)}$$

$$g(BAH) = (BAH - t1)^3_+ - (BAH - t2)^3_+ \frac{(t3-t1)}{(t3-t2)} + (BAH - t3)^3_+ \frac{(t2-t1)}{(t3-t2)}$$

$$= (5.37898 - 0.193)^3_+ - (5.37898 - 0.871)^3_+ \frac{(9.294-0.193)}{(9.294-0.871)} + 0$$, since X< t3

$$= (5.18598)^3_+ - (4.50798)^3_+ \frac{(9.101)}{(8.423)} + 0$$

$$= 139.47376 - 91.61065 * 1.08049 + 0$$

$$= 139.47376 - 98.98438 + 0$$

$$= 40.48937$$

Hence, the volume predicted by model 16 for this tree is;

$$V = \beta_0 + \beta_1 . BAH.m3 + \beta_2 BAH.m3_2 + \varepsilon$$

$$= -0.0009352 + 0.4284381 * 5.37898 + (-0.0016409 * 40.48937$$

$$= -0.0009352 + 2.30456 + (-0.06644)$$

$$= -0.02511 + 3.05 - 0.24$$

= **2.23718 m^3**

The field measured volume for this particular tree with DBH of 54.1 cm and height of 23.4 m is 2.10895 m^3.

11. Model Performance

To assess the performance of selected models, we compared the volume predicted by selected models (7 and 16) with the volume of the tree as measured in the field.
Using the equations of the selected model, volume prediction or estimation was done in R.

SN	Tree_ID	Height (in m)	DBH (in cm)	Volume in m^3 (Field measured) [A]	Predicted Volume Model_16 [B]	Predicted Volume Model_7 [C]	Difference (Field – Model_16) [A – B]	Difference (Field – Model7) [A – C]
1	pwec01	10.9	14.5	0.078113	0.07618366	0.07113195	0.001929062	0.00698077
2	pwec02	33.33	37	1.869446	1.56272007	0.89112436	0.306726082	0.978321795
3	pwec03	25.2	27	0.822746	0.62060844	0.36224004	0.202137266	0.460505664
4	pwec04	35.2	69.8	4.565907	6.07348901	6.02279896	-1.5075817	-1.456891657
5	pwec05	26.5	28.5	0.901824	0.72809198	0.41792703	0.173731862	0.483896807
6	pwec07	13.2	12	0.049595	0.06290208	0.03661279	-0.013307557	0.01298173
7	pwec09	8.5	10	0.021253	0.02777015	0.01404257	-0.006517341	0.007210242
8	pwec10	34.65	71.7	6.719218	6.31390729	6.45985285	0.405310616	0.259365055
9	pwec11	37.2	65.8	5.640111	5.69726193	5.14456805	-0.057151043	0.495542839
10	pwec12	33.3	57.2	4.599075	3.82076732	3.44354537	0.778307495	1.155529448
11	pwec13	27.82	40	1.466955	1.52439037	1.134087	-0.057435192	0.332868179
12	pwec14	31.7	73.5	6.325424	6.06523036	6.88247571	0.260193169	-0.557052176
13	pwec15	16.25	15.3	0.146149	0.12716975	0.08374472	0.018978841	0.062403867
14	pwec16	26.56	60.6	3.907716	3.40927637	4.08153465	0.49843948	-0.173818801
15	pwec18	31.53	62.7	4.277388	4.36119698	4.50135735	-0.083809284	-0.223969654
16	pwec19	51.5	95.8	16.71401	16.925805	12.9950454	-0.211796365	3.718963277
17	pwec20	28.1	30.3	0.899288	0.87445435	0.4948257	0.024833864	0.404462521
18	pwec21	32.55	55.7	4.372989	3.53383112	3.17879315	0.839157927	1.1941959
19	pwec22	17.92	21.9	0.321093	0.28887385	0.21402937	0.032219083	0.107063566
20	pwec23	11.28	17	0.118464	0.10874536	0.1122971	0.009718536	0.006166803
21	pwec24	18.82	31.3	0.740416	0.62191166	0.54219749	0.118504779	0.198218946
22	pwec25	46.81	71.2	8.161811	8.44647249	6.34440465	-0.284661339	1.8174065
23	pwec29	23.69	35.6	1.157502	1.0197041	0.7923857	0.137797751	0.365116158
24	pwec30	26.51	24.1	0.634743	0.51869893	0.27085812	0.116044089	0.363884904
25	pwec31	25.85	42.5	2.270154	1.60063696	1.37026194	0.669517232	0.899892254
26	pwec32	15.27	13.3	0.07656	0.08989369	0.05387237	-0.013333737	0.022687581
27	pwec35	30.3	47.6	2.941106	2.37641092	1.95548729	0.564694683	0.985618315
28	pwec36	34.54	66.7	5.779715	5.43115009	5.33835609	0.348565269	0.441359271
29	pwwc01	61.3	78.5	15.19106	13.5067267	8.11323594	1.684332075	7.077822851
30	pwwc02	50.5	65.5	7.88437	7.70227698	5.0827208	0.182092608	2.801648786
31	pwwc03	47.4	49	3.997815	3.99598728	2.14078412	0.00182802	1.857031188
32	pwwc04	25.6	35.8	1.294803	1.11702065	0.80686972	0.17778211	0.487933038
33	pwwc05	43.4	57.5	5.890892	5.06547572	3.49776959	0.825416279	2.393122411
34	pwwc06	20.5	28	0.626011	0.5420855	0.39863235	0.083925786	0.227378929
35	pwwc07	15.5	19.4	0.188015	0.19574877	0.15843918	-0.007734217	0.029575378

36	pww01	40.4	68.2	7.114065	6.66535844	5.6661465	0.448706754	1.44791869
37	pww02	32.3	46.7	2.516509	2.43983055	1.84129864	0.076678116	0.67521002
38	pww03	34.3	59.8	4.324318	4.31485682	3.92792378	0.00946109	0.396394123
39	pww04	29.7	29.5	1.001794	0.87576803	0.45882094	0.126026366	0.542973452
40	pww05	20.6	16.9	0.220556	0.19660635	0.11030332	0.023949919	0.110252951
41	pww06	25.6	35.2	1.158843	1.07867902	0.76613609	0.080164297	0.392707221
42	pww07	46.7	79.5	10.20937	10.5304035	8.36887123	-0.321036872	1.840495361
43	pww09	30	54	3.680908	3.04811869	2.8915313	0.632789093	0.789376481
44	pww10	35	79	9.103551	7.76605205	8.24105359	1.337499233	0.862497701
45	pww11	24	37.2	1.094427	1.13069237	0.90637161	-0.036265778	0.188054982
46	pww12	29	49.5	2.578732	2.46113463	2.20852194	0.117597108	0.370209791
47	pww13	31	45.6	5.977509	2.22671373	1.70812023	3.750795642	4.269389144
48	pww14	37	75.5	6.455855	7.49443445	7.36488424	-1.038579092	-0.909028879
49	pww15	32.2	59.8	3.714819	4.04415888	3.92792378	-0.329339783	-0.213104683
50	pww16	31.3	51.4	2.861488	2.87707876	2.48398722	-0.015590539	0.377501002
51	pww17	32.4	58.9	3.545912	3.94506641	3.75665971	-0.399154696	-0.210747996
52	pww18	31.6	51.8	2.866773	2.95138824	2.54357308	-0.084614911	0.323200249
53	pww19	29.7	44.7	2.532482	2.04540316	1.60418333	0.48707897	0.928298796
54	pww20	31.2	46.4	2.331334	2.32345663	1.80432376	0.007877627	0.527010493
55	pww21	24.6	53.3	2.544645	2.41943919	2.77808205	0.125206013	-0.233436847
56	pww22	24.3	56.8	2.484116	2.72363579	3.37148302	-0.239520085	-0.887367323
57	pww23	23.4	63.8	2.826222	3.32735963	4.72606834	-0.501137471	-1.899846181
58	pww24	25	42.3	0.980704	1.53151892	1.34915523	-0.550814891	-0.368451197
59	pww25	25	38.7	0.970184	1.27698373	1.02290491	-0.306799478	-0.05272066
60	pww26	21	47	1.033628	1.58948157	1.87853335	-0.5558537	-0.844905476
61	pww27	23	57	2.046356	2.59272602	3.40747946	-0.546369772	-1.361123211
62	pww28	23	56.8	2.759748	2.57365337	3.37148302	0.186094459	-0.611735195
63	pww29	23	43.8	1.747233	1.51058216	1.50580637	0.236650804	0.241426592
64	pww30	26	48.4	2.333366	2.1012689	2.05969636	0.232097067	0.273669606
65	pww31	25	57.2	2.908168	2.8451846	3.44354537	0.062983867	-0.535376909
66	pww32	26	57.2	2.868785	2.96233429	3.44354537	-0.093549174	-0.574760257
67	pww33	26	58.5	2.847817	3.10245381	3.68153964	-0.254636481	-0.833722313
68	pww34	22	70	2.91828	3.7785788	6.06815361	-0.860298516	-3.149873331
69	pww35	21	39	0.850265	1.08704901	1.04855448	-0.236783915	-0.198289387
70	pww36	14	40.2	0.880749	0.76560228	1.15096067	0.115146919	-0.270211475
71	pww37	23	50	1.540081	1.98014801	2.27872644	-0.440066951	-0.738645376
72	pww38	25	56	2.244867	2.72318087	3.23021783	-0.47831378	-0.985350737
73	pww39	14	30	0.452232	0.42404626	0.48140544	0.028185768	-0.02917342
74	pww40	13.5	24.6	0.310036	0.27384116	0.28504059	0.036194625	0.024995195
75	pww41	12	27	0.338292	0.29402923	0.36224004	0.044262749	-0.023948061
76	pww42	42	53	2.943593	4.14509582	2.73036042	-1.201502427	0.213232975
77	pww43	47	49	3.155671	3.9611221	2.14078412	-0.805451325	1.014886658
78	pww44	47	52	2.700542	4.47452392	2.5753894	-1.773982249	0.12515227
79	pww45	7	20	0.094459	0.09332121	0.17062117	0.001137836	-0.076162125
80	pww46	9.5	18.7	0.109302	0.11088766	0.144318	-0.00158583	-0.035016166

81	pww47	13	28	0.382067	0.34261252	0.39863235	0.039454438	-0.016565394
82	pww48	7	20.9	0.066456	0.10189011	0.19043562	-0.035434055	-0.123979557
83	pww49	8.7	16	0.074873	0.07404147	0.09502983	0.000831676	-0.020156687
84	pww50	6.6	17	0.062234	0.06333052	0.1122971	-0.001096893	-0.050063474
85	pww51	8	17.6	0.071559	0.08218179	0.12294174	-0.010622761	-0.051382708
86	pww52	8.2	15	0.063951	0.06118832	0.07909791	0.0027628	-0.015146786
87	pww53	11	22.3	0.192571	0.18331503	0.22387854	0.009256463	-0.031307049
88	pww54	13	21.7	0.193346	0.20518272	0.20913567	-0.011836439	-0.01578938
89	pww55	8	17	0.09702	0.07704053	0.1122971	0.019979657	-0.015276905
90	pww56	6.4	19.6	0.095293	0.08175335	0.16249107	0.013539838	-0.067197879
91	pww57	11	24.8	0.211825	0.22662879	0.29108082	-0.01480408	-0.079256111
92	pww58	9	17	0.1067	0.08646617	0.1122971	0.02023338	-0.005597541
93	pww59	11	22	0.20019	0.17817062	0.21613284	0.022019455	-0.01594277
94	pww60	6	12	0.038345	0.02819859	0.03661279	0.010146461	0.001732262
95	pww61	8	19.6	0.131715	0.10274701	0.16249107	0.028968137	-0.03077592
96	pww62	8.5	17	0.082988	0.08175335	0.1122971	0.001234727	-0.029309016
97	pww63	10	15.5	0.099444	0.0800396	0.08706387	0.019404592	0.012380323
98	pww64	10.3	20	0.156791	0.13745391	0.17062117	0.019337082	-0.013830181
99	pww65	13	29	0.331729	0.36757786	0.43859024	-0.035848521	-0.106860901
100	pww66	7.5	13	0.061002	0.04190861	0.04988939	0.019093583	0.011112803
101	pww67	9.2	22.8	0.205234	0.15973898	0.23595679	0.045494953	-0.030722854
102	pww68	12.3	21.4	0.235203	0.18888852	0.20217832	0.046314478	0.033024675
103	pww69	16	28	0.604173	0.42232035	0.39863235	0.181852506	0.205540504
104	pww70	15.4	23	0.348048	0.27298232	0.24097046	0.075065484	0.107077341
105	pww71	8.4	16	0.07975	0.07147084	0.09502983	0.00827947	-0.015279522
106	pww72	10	16.3	0.10968	0.08860837	0.10034075	0.021071713	0.009339328
107	pww73	5.6	11.4	0.029391	0.02348577	0.02931066	0.005905021	8.01325E-05
108	pww74	8.2	12	0.041698	0.03890954	0.03661279	0.00278895	0.005085704
109	pww75	10.4	23.3	0.202828	0.18888852	0.24889928	0.0139399	-0.046070864
110	pww76	4.6	17.5	0.050416	0.04662143	0.12160994	0.003794215	-0.0711943
111	pww77	9	21	0.162673	0.13231175	0.19250049	0.030360861	-0.029827887
112	pww78	5.4	12	0.024885	0.02519952	0.03661279	-0.000315024	-0.01172829
113	pww79	8.6	23	0.184183	0.15202444	0.24097046	0.032158462	-0.056787562
114	pww80	5.6	16	0.062674	0.04747831	0.09502983	0.015195584	-0.032355941
115	pww81	10	30	0.324308	0.30219337	0.48140544	0.022114666	-0.157097412
116	pww82	9.3	17	0.099659	0.08946525	0.1122971	0.010194073	-0.012637775
117	pww83	10.6	20.3	0.098011	0.14602461	0.17742547	-0.048013628	-0.079414493
118	pww84	8.2	17.6	0.073422	0.08432398	0.12294174	-0.010901755	-0.049519511
119	pww86	10	23	0.162825	0.17688456	0.24097046	-0.014059315	-0.078145214
120	pww87	6.4	16	0.054447	0.05433331	0.09502983	0.000114089	-0.040582427
121	pww88	9	17.9	0.108497	0.09632031	0.12894007	0.012176892	-0.020442864
122	pww89	9.2	20	0.129081	0.12288486	0.17062117	0.006196534	-0.041539778
123	pww90	11.3	29.2	0.451779	0.32368683	0.44681162	0.12809225	0.00496746
124	pww91	11	16	0.096578	0.09374966	0.09502983	0.002827913	0.001547738
125	pww92	15	22	0.244287	0.24336244	0.21613284	0.000925046	0.028154644

126	pww93	16	23	0.308507	0.28371915	0.24097046	0.024788047	0.067536734
127	pww94	13.4	22	0.187798	0.21719157	0.21613284	-0.029393862	-0.028335133
128	pww95	9	16	0.081961	0.0766121	0.09502983	0.005348773	-0.013068962
129	pww96	12	26	0.281568	0.2721235	0.32825538	0.009444034	-0.04668785
130	pww97	10.5	16.2	0.114164	0.09160745	0.09834906	0.022556945	0.01581533
131	pww98	12	25	0.235995	0.25151674	0.29716349	-0.015521993	-0.061168746
132	pww99	11.2	24.5	0.176213	0.22534181	0.28203602	-0.049128461	-0.105822672
133	pww100	10.5	19.6	0.124619	0.13488281	0.16249107	-0.010264056	-0.03787232
134	pww101	15.7	26	0.361353	0.35681435	0.32825538	0.004538227	0.033097195
135	pww102	10.4	19.4	0.140351	0.13102623	0.15843918	0.009324901	-0.018088045
136	pww103	12	30	0.296515	0.36284142	0.48140544	-0.066326353	-0.184890377
137	pww104	12.6	30.8	0.349919	0.40204886	0.51824987	-0.052129415	-0.168330424
138	pww105	12.4	33	0.393391	0.45426661	0.63222713	-0.060875914	-0.238836425
139	pww106	14	32	0.455913	0.48278939	0.57804844	-0.026876615	-0.122135668
140	pww107	8.2	15	0.046179	0.06118832	0.07909791	-0.015009287	-0.032918872
141	pww108	15	42.3	0.824048	0.91037999	1.34915523	-0.086332085	-0.525107328
142	pww109	12.4	21.3	0.154964	0.18803103	0.19940601	-0.03306681	-0.04444179
143	pww110	10.5	17.3	0.096358	0.10488927	0.11761679	-0.0085315	-0.021259024
144	pww111	14	21.4	0.23057	0.21504696	0.20217832	0.015523239	0.02839188
145	pww112	9.3	13	0.040927	0.05219112	0.04988939	-0.011264325	-0.008962591
146	pww113	21	83.6	6.108012	5.18293198	9.45119807	0.925079897	-3.34318619
147	pww114	14	50	0.971455	1.19205686	2.27872644	-0.220602092	-1.307271665
148	pww115	20	61	2.202325	2.58091857	4.15956078	-0.378593578	-1.957235781
149	pww116	21.3	54.6	1.588924	2.192393	2.99048431	-0.603469395	-1.401560708
150	pww117	17	43.7	1.145585	1.10467639	1.49484635	0.040908336	-0.349261628
151	pww118	21.2	53	1.757803	2.05305886	2.73036042	-0.295256326	-0.972557881
152	pww119	19.3	60.4	1.786667	2.43801774	4.04256729	-0.651351026	-2.255900577
153	pww120	18.8	51	1.455383	1.67790798	2.4248086	-0.222525472	-0.969426099
154	pww121	16.3	31.3	0.441257	0.53775325	0.54219749	-0.096496033	-0.100940275
155	pww122	16	26.3	0.29933	0.37188441	0.33781809	-0.072554175	-0.038487858
156	pww123	14	22.3	0.201306	0.23349323	0.22387854	-0.032187481	-0.022572785
157	pww124	15.8	34.5	0.607652	0.63494688	0.72173433	-0.02729534	-0.114082781
158	pww125	16.6	28.3	0.378011	0.44735627	0.40997795	-0.069345255	-0.031966933
159	pww126	34	128	15.89807	19.9677386	24.6635594	-4.069670478	-8.765491335
160	pww127	29.3	94	8.904098	9.22370258	12.4425433	-0.319604644	-3.538445349
161	pww128	28.5	78.2	6.396678	6.17488679	8.03695767	0.221790959	-1.640279915
162	pww129	24	58	2.569473	2.80692633	3.5884589	-0.237453578	-1.018986142
163	pww130	23.4	46.2	1.456198	1.71413159	1.77926467	-0.25793328	-0.323066357
164	pww131	16	23	0.265894	0.28371915	0.24097046	-0.017824815	0.024923873
165	pww132	25	64.2	3.603007	3.60713695	4.80853133	-0.004129504	-1.205523889
166	pww133	23.3	47.5	1.831582	1.80682897	1.94172995	0.024752691	-0.110148284
167	pww134	22	44.5	1.632743	1.49054589	1.58176695	0.142197121	0.050976062
168	pww135	24.7	62.5	3.411441	3.37174484	4.46012992	0.039695759	-1.048689318
169	pww136	15	20	0.189707	0.2008944	0.17062117	-0.011187336	0.01908589
170	pww137	24.3	65.4	3.671695	3.63921611	5.06004347	0.032478719	-1.388348648

171	pww138	26	55	2.788948	2.73273461	3.05884104	0.056212993	-0.269893437
172	pww139	25.6	78	5.732931	5.50639551	7.98541829	0.226535167	-2.252487621
173	pww140	21	59	2.779737	2.53416121	3.77496352	0.245575794	-0.995226512
174	pww141	17.4	38.4	0.841345	0.869638	0.99885274	-0.028292582	-0.157507323
175	pww142	13	29.2	0.414731	0.3727458	0.44681162	0.041985363	-0.032080458
176	pww143	24	84.5	6.059841	6.06935969	9.69652548	-0.009518376	-3.636684174
177	pww144	16	36	0.741473	0.70107728	0.82024898	0.040395333	-0.078776365
178	pww145	22.8	72.5	4.276918	4.2125414	6.64539459	0.064376609	-2.368476581
179	pww146	25.8	84.4	6.911923	6.51716167	9.66972501	0.394760916	-2.75780242
180	pww147	11.3	17	0.101216	0.10917382	0.1122971	-0.007958238	-0.011081512
181	pww148	13	19.9	0.18094	0.17216921	0.1685852	0.008771174	0.01235518
182	pww149	17	42	1.125652	1.01882463	1.3192258	0.106827616	-0.193573562
183	pww150	11	13.7	0.061415	0.06847177	0.05918301	-0.007056394	0.002232368
184	pww151	12	24.2	0.265914	0.23563855	0.27382537	0.030275212	-0.007911602
185	pww152	14	23	0.241889	0.24808318	0.24097046	-0.006193754	0.000918965
186	pww153	9	18.3	0.104176	0.10060477	0.13628494	0.003571207	-0.032108959
187	pww154	14	22	0.199255	0.2270578	0.21613284	-0.027802933	-0.01687798
188	pww155	16	20.2	0.17615	0.21847837	0.1747004	-0.042328613	0.001449363
189	pww156	12.2	23.6	0.175945	0.2274868	0.25689217	-0.051541415	-0.080946786
190	pww157	10	16	0.074455	0.08518086	0.09502983	-0.010726199	-0.020575171
191	pww158	12	15.6	0.086466	0.0971772	0.08839153	-0.010711178	-0.001925507
192	pww159	14	24.1	0.248268	0.27255291	0.27085812	-0.024284802	-0.022590012
193	pww160	11.2	16.5	0.103931	0.10189011	0.10366069	0.002040392	0.000269815
194	pww161	12.2	25	0.235723	0.25580904	0.29716349	-0.020086254	-0.061440709
195	pww162	13	24.4	0.235089	0.25967245	0.27978926	-0.024583393	-0.044700208
196	pww163	16	40.7	0.73891	0.89898477	1.19645334	-0.160074667	-0.457543243
197	pww164	13.6	23	0.242572	0.24078768	0.24097046	0.001784227	0.001601444
198	pww165	13	24.7	0.2279	0.2661122	0.28805548	-0.038212452	-0.060155732
199	pww166	11.3	25	0.190179	0.23692579	0.29716349	-0.046747016	-0.106984722
200	pww167	14.2	26.2	0.335669	0.32712704	0.33461786	0.008542414	0.001051592
201	pww168	16.1	27	0.383657	0.39515125	0.36224004	-0.011494594	0.021416618
202	pww169	12.4	22.4	0.161215	0.20861359	0.22600027	-0.047399	-0.064785682
203	pww170	12	20	0.12753	0.16059619	0.17062117	-0.033065988	-0.043090977
204	pww171	15.2	20.9	0.201818	0.22233895	0.19043562	-0.020520988	0.01138235
205	pww172	15	25.8	0.268665	0.33529899	0.32194253	-0.066633607	-0.053277143
206	pww173	12	16.8	0.109133	0.11302997	0.10897442	-0.00389731	0.000158244
207	pww174	15.3	28.9	0.440642	0.43008776	0.43405573	0.010554566	0.00658659
208	pww175	16	27.8	0.415747	0.41584914	0.39087252	-0.000102346	0.024874276
209	pww176	17.7	28.5	0.474469	0.48408655	0.41792703	-0.009617122	0.056542394
210	pww177	19.5	39.6	0.771023	1.0394982	1.09926362	-0.268474888	-0.328240305
211	pww178	20.1	30.8	0.615584	0.64364013	0.51824987	-0.028055695	0.097334574
212	pww179	23.6	43	1.48927	1.49321685	1.42051549	-0.003947347	0.068754012
213	pww180	16.3	24	0.346377	0.31508785	0.26790046	0.031288693	0.078476079
214	pww181	19	31.8	0.750342	0.64885727	0.56769383	0.101484821	0.182648266
215	pww182	12	12.8	0.064122	0.06547271	0.04723407	-0.001350844	0.016887792

216	pww183	11.3	14.7	0.055135	0.08132492	0.0744511	-0.026190126	-0.019316311
217	pww184	10.5	11	0.018133	0.04190861	0.02466385	-0.023775346	-0.006530586
218	pww185	12	18.9	0.156072	0.14345334	0.14834376	0.012619026	0.007728604
219	pww186	17.5	20.1	0.302507	0.23692579	0.17265955	0.065580832	0.129847066
220	pww187	15.2	20	0.211935	0.20346736	0.17062117	0.008467921	0.041314107
221	pww188	16.7	19.3	0.17588	0.20861359	0.1564163	-0.032733669	0.019463619
222	pww189	13.2	19.8	0.169681	0.17345519	0.16655157	-0.00377422	0.003129401
223	pww190	17.4	23	0.324292	0.30864002	0.24097046	0.015651683	0.083321239
224	pww191	14.4	16	0.111178	0.12288486	0.09502983	-0.011706922	0.01614811
225	pww192	12.7	15.3	0.076498	0.09931943	0.08374472	-0.02282161	-0.007246902
226	pww193	17.4	23.4	0.32618	0.31981702	0.25179821	0.006362807	0.074381615
227	pww194	18	25	0.414664	0.37834547	0.29716349	0.036318177	0.117500158
228	pww195	14.3	19.2	0.131714	0.17688456	0.15439538	-0.0451702	-0.022681015
229	pww196	13	19.4	0.143229	0.16402508	0.15843918	-0.02079622	-0.015210314
230	pww197	15	22	0.233531	0.24336244	0.21613284	-0.009830985	0.017398613
231	pww198	13.2	17.1	0.107499	0.12931223	0.11429144	-0.021813149	-0.006792366
232	pww199	14	19	0.152504	0.1695973	0.15035918	-0.01709378	0.002144344
233	pww200	11.2	15.2	0.060178	0.08603774	0.08175323	-0.025860146	-0.02157564
234	pww201	12	16	0.09261	0.10231856	0.09502983	-0.009709013	-0.00242028
235	pww202	17.3	31	0.587155	0.56072141	0.52816473	0.026433941	0.058990618
236	pww203	16	25	0.352678	0.33615932	0.29716349	0.016518415	0.055514243
237	pww204	16.3	26.2	0.340898	0.37619162	0.33461786	-0.03529379	0.006279966
238	pww205	12.2	16	0.099661	0.10403237	0.09502983	-0.004371473	0.004631061
239	pww206	17.3	28.3	0.467506	0.46636373	0.40997795	0.001142043	0.057527816
240	pww207	16.1	30.2	0.570543	0.49446593	0.49001204	0.076077445	0.080531338
241	pww208	20.4	37.3	0.949635	0.96390106	0.91403725	-0.014266499	0.035597303
242	pww209	19.3	34.7	0.789986	0.78699416	0.73446429	0.00299196	0.055521834
243	pww210	16.4	31	0.534297	0.53125607	0.52816473	0.003040524	0.006131858
244	pww211	17	24.4	0.394784	0.34046131	0.27978926	0.054323084	0.114995128
245	pww212	20.3	43.4	1.305046	1.30444795	1.46214871	0.000597677	-0.157103083
246	pww213	19.4	37	0.90311	0.9002994	0.89112436	0.002810873	0.011985921
247	pww214	19.2	35.9	0.498251	0.83813004	0.81293911	-0.33987928	-0.314688345
248	pww215	20.5	43.1	1.183106	1.29957386	1.43126499	-0.116468251	-0.248159387
249	pww216	23.4	54.1	2.108954	2.370525	2.9089355	-0.261571239	-0.799981737
				378.2779	**381.139206**	**398.699353**	**-2.86127217**	**-20.42141925**

From the above table, the difference [A-B] provides difference between the volume measured in the field (actual volume) and the volume predicted by model 16. The figures with negative (-) indicates that the volume has been over-predicted by the model 16 vis-à-vis actual volume of the particular tree. And the figures without negative (-) sign indicates the under prediction of volume by the model 16.

Similarly, the difference [A-C] is the difference between the actual volume and the volume predicted by the model 7. Same explanation is applicable here – the figures with negative sign indicates overprediction of volume by the model and vice-versa, while those figures without (-) are under prediction of volume by the model 7.

Summation of the figures in the difference column results in -2.86127 and -20.42141 for model 16 and model 7 respectively. These indicate that the model 16 over predicts total volume for 249 trees by only 2.86127 m^3, while the model 7 over predicts the total volume of 249 trees by 20.42141 m^3. Therefore, looking at this, one may be inclined to conclude that overall, model 16 predicts better than model 7.

12. Limitations of the model

The model has the following limitations;

1. The modeling has been done based on only 249 sample trees. Despite the best attempts made, the data collection is more confined to western region of the country. Out of 249 trees sampled, 214 sample trees have been felled from Thimphu, Paro and Haa dzongkhags.

2. The diameter for the samples ranges between minimum of 10 cm to 128 cm (over bark). Thus, use the model for predicting volume above 128 cm should be done with caution since there were very limited sample trees in larger diameter class.

13. Conclusion

The model 16 which uses the height outperforms the model 7 which doesn't use the height, as empirically shown above. This further reinforces and confirms the observations made by Professor Timothy Gordon Gregoire and Mr. Yograj Chettri while modeling broadleaf species for biomass estimation. They too observed that especially in conifers, the models fitted with predictors with height predicted the biomass better than those models that didn't use height as predictor variable.

This, therefore, leads us to confidently conclude that the best model for *Pinus wallichiana*, out of 16 models fitted above is model 16. But since the models have been developed with and without height as predictors, we considered two models as best fit model;

1. Model 7 - the best fit model which doesn't use height
2. Model 16 - the best fit model which uses the height

14. Acknowledgement

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16. Annexure – Dataset for Pinus wallichiana

SN	Tree_ID	Height.m	DBH.cm	Volume.m3	BA.m2	BAH.m3	DBH2H.m3
1	pwec01	10.9	14.5	0.078113	0.016513	0.179992	0.229173
2	pwec02	33.33	37	1.869446	0.107521	3.583675	4.562877
3	pwec03	25.2	27	0.822746	0.057256	1.442839	1.83708
4	pwec04	35.2	69.8	4.565907	0.382649	13.46925	17.14958
5	pwec05	26.5	28.5	0.901824	0.063794	1.69054	2.152463
6	pwec07	13.2	12	0.049595	0.01131	0.149288	0.19008
7	pwec09	8.5	10	0.021253	0.007854	0.066759	0.085
8	pwec10	34.65	71.7	6.719218	0.403765	13.99044	17.81318
9	pwec11	37.2	65.8	5.640111	0.340049	12.64983	16.10626
10	pwec12	33.3	57.2	4.599075	0.25697	8.557091	10.89523
11	pwec13	27.82	40	1.466955	0.125664	3.495964	4.4512
12	pwec14	31.7	73.5	6.325424	0.424292	13.45005	17.12513
13	pwec15	16.25	15.3	0.146149	0.018385	0.298763	0.380396
14	pwec16	26.56	60.6	3.907716	0.288426	7.660607	9.753788
15	pwec18	31.53	62.7	4.277388	0.308763	9.735291	12.39536
16	pwec19	51.5	95.8	16.71401	0.72081	37.12172	47.26485
17	pwec20	28.1	30.3	0.899288	0.072107	2.026196	2.579833
18	pwec21	32.55	55.7	4.372989	0.243669	7.931426	10.09861
19	pwec22	17.92	21.9	0.321093	0.037668	0.675019	0.859461
20	pwec23	11.28	17	0.118464	0.022698	0.256034	0.325992
21	pwec24	18.82	31.3	0.740416	0.076945	1.448099	1.843777
22	pwec25	46.81	71.2	8.161811	0.398153	18.63754	23.73005
23	pwec29	23.69	35.6	1.157502	0.099538	2.35806	3.002376
24	pwec30	26.51	24.1	0.634743	0.045617	1.209299	1.539727
25	pwec31	25.85	42.5	2.270154	0.141863	3.667147	4.669156
26	pwec32	15.27	13.3	0.07656	0.013893	0.212145	0.270111
27	pwec35	30.3	47.6	2.941106	0.177952	5.391957	6.865253
28	pwec36	34.54	66.7	5.779715	0.349415	12.06879	15.36647
29	pwwc01	61.3	78.5	15.19106	0.483982	29.6681	37.77459
30	pwwc02	50.5	65.5	7.88437	0.336955	17.01625	21.66576
31	pwwc03	47.4	49	3.997815	0.188574	8.938412	11.38074
32	pwwc04	25.6	35.8	1.294803	0.10066	2.57689	3.280998
33	pwwc05	43.4	57.5	5.890892	0.259672	11.26978	14.34913
34	pwwc06	20.5	28	0.626011	0.061575	1.262292	1.6072
35	pwwc07	15.5	19.4	0.188015	0.029559	0.458168	0.583358
36	pww01	40.4	68.2	7.114065	0.365308	14.75842	18.79101
37	pww02	32.3	46.7	2.516509	0.171287	5.53256	7.044275
38	pww03	34.3	59.8	4.324318	0.280862	9.63355	12.26582
39	pww04	29.7	29.5	1.001794	0.068349	2.029973	2.584643

40	pww05	20.6	16.9	0.220556	0.022432	0.462094	0.588357
41	pww06	25.6	35.2	1.158843	0.097314	2.491238	3.171942
42	pww07	46.7	79.5	10.20937	0.496391	23.18147	29.51557
43	pww09	30	54	3.680908	0.229022	6.870663	8.748
44	pww10	35	79	9.103551	0.490167	17.15584	21.8435
45	pww11	24	37.2	1.094427	0.108687	2.608477	3.321216
46	pww12	29	49.5	2.578732	0.192442	5.580823	7.105725
47	pww13	31	45.6	5.977509	0.163313	5.062689	6.446016
48	pww14	37	75.5	6.455855	0.447697	16.56477	21.09093
49	pww15	32.2	59.8	3.714819	0.280862	9.043741	11.51485
50	pww16	31.3	51.4	2.861488	0.207499	6.49472	8.269335
51	pww17	32.4	58.9	3.545912	0.272471	8.828064	11.24024
52	pww18	31.6	51.8	2.866773	0.210741	6.659421	8.479038
53	pww19	29.7	44.7	2.532482	0.15693	4.66081	5.934327
54	pww20	31.2	46.4	2.331334	0.169093	5.275704	6.717235
55	pww21	24.6	53.3	2.544645	0.223123	5.488825	6.988589
56	pww22	24.3	56.8	2.484116	0.253388	6.157336	7.839763
57	pww23	23.4	63.8	2.826222	0.319692	7.480784	9.52483
58	pww24	25	42.3	0.980704	0.140531	3.513263	4.473225
59	pww25	25	38.7	0.970184	0.117628	2.940707	3.744225
60	pww26	21	47	1.033628	0.173494	3.643384	4.6389
61	pww27	23	57	2.046356	0.255176	5.869045	7.4727
62	pww28	23	56.8	2.759748	0.253388	5.827931	7.420352
63	pww29	23	43.8	1.747233	0.150674	3.4655	4.412412
64	pww30	26	48.4	2.333366	0.183984	4.78359	6.090656
65	pww31	25	57.2	2.908168	0.25697	6.424243	8.1796
66	pww32	26	57.2	2.868785	0.25697	6.681213	8.506784
67	pww33	26	58.5	2.847817	0.268783	6.988355	8.89785
68	pww34	22	70	2.91828	0.384845	8.466592	10.78
69	pww35	21	39	0.850265	0.119459	2.50864	3.1941
70	pww36	14	40.2	0.880749	0.126923	1.776929	2.262456
71	pww37	23	50	1.540081	0.19635	4.516039	5.75
72	pww38	25	56	2.244867	0.246301	6.157522	7.84
73	pww39	14	30	0.452232	0.070686	0.989602	1.26
74	pww40	13.5	24.6	0.310036	0.047529	0.641644	0.816966
75	pww41	12	27	0.338292	0.057256	0.687066	0.8748
76	pww42	42	53	2.943593	0.220618	9.26597	11.7978
77	pww43	47	49	3.155671	0.188574	8.862983	11.2847
78	pww44	47	52	2.700542	0.212372	9.981468	12.7088
79	pww45	7	20	0.094459	0.031416	0.219911	0.28
80	pww46	9.5	18.7	0.109302	0.027465	0.260914	0.332206
81	pww47	13	28	0.382067	0.061575	0.800478	1.0192

82	pww48	7	20.9	0.066456	0.034307	0.240149	0.305767
83	pww49	8.7	16	0.074873	0.020106	0.174924	0.22272
84	pww50	6.6	17	0.062234	0.022698	0.149807	0.19074
85	pww51	8	17.6	0.071559	0.024328	0.194628	0.247808
86	pww52	8.2	15	0.063951	0.017671	0.144906	0.1845
87	pww53	11	22.3	0.192571	0.039057	0.429628	0.547019
88	pww54	13	21.7	0.193346	0.036984	0.480787	0.612157
89	pww55	8	17	0.09702	0.022698	0.181584	0.2312
90	pww56	6.4	19.6	0.095293	0.030172	0.1931	0.245862
91	pww57	11	24.8	0.211825	0.048305	0.531356	0.676544
92	pww58	9	17	0.1067	0.022698	0.204282	0.2601
93	pww59	11	22	0.20019	0.038013	0.418146	0.5324
94	pww60	6	12	0.038345	0.01131	0.067858	0.0864
95	pww61	8	19.6	0.131715	0.030172	0.241375	0.307328
96	pww62	8.5	17	0.082988	0.022698	0.192933	0.24565
97	pww63	10	15.5	0.099444	0.018869	0.188692	0.24025
98	pww64	10.3	20	0.156791	0.031416	0.323584	0.412
99	pww65	13	29	0.331729	0.066052	0.858676	1.0933
100	pww66	7.5	13	0.061002	0.013273	0.099549	0.12675
101	pww67	9.2	22.8	0.205234	0.040828	0.375619	0.478253
102	pww68	12.3	21.4	0.235203	0.035968	0.442408	0.563291
103	pww69	16	28	0.604173	0.061575	0.985203	1.2544
104	pww70	15.4	23	0.348048	0.041548	0.639832	0.81466
105	pww71	8.4	16	0.07975	0.020106	0.168892	0.21504
106	pww72	10	16.3	0.10968	0.020867	0.208672	0.26569
107	pww73	5.6	11.4	0.029391	0.010207	0.057159	0.072778
108	pww74	8.2	12	0.041698	0.01131	0.09274	0.11808
109	pww75	10.4	23.3	0.202828	0.042638	0.44344	0.564606
110	pww76	4.6	17.5	0.050416	0.024053	0.110643	0.140875
111	pww77	9	21	0.162673	0.034636	0.311725	0.3969
112	pww78	5.4	12	0.024885	0.01131	0.061073	0.07776
113	pww79	8.6	23	0.184183	0.041548	0.357309	0.45494
114	pww80	5.6	16	0.062674	0.020106	0.112595	0.14336
115	pww81	10	30	0.324308	0.070686	0.706858	0.9
116	pww82	9.3	17	0.099659	0.022698	0.211091	0.26877
117	pww83	10.6	20.3	0.098011	0.032365	0.343074	0.436815
118	pww84	8.2	17.6	0.073422	0.024328	0.199494	0.254003
119	pww86	10	23	0.162825	0.041548	0.415476	0.529
120	pww87	6.4	16	0.054447	0.020106	0.12868	0.16384
121	pww88	9	17.9	0.108497	0.025165	0.226484	0.288369
122	pww89	9.2	20	0.129081	0.031416	0.289027	0.368
123	pww90	11.3	29.2	0.451779	0.066966	0.756718	0.963483

124	pww91	11	16	0.096578	0.020106	0.221168	0.2816
125	pww92	15	22	0.244287	0.038013	0.570199	0.726
126	pww93	16	23	0.308507	0.041548	0.664761	0.8464
127	pww94	13.4	22	0.187798	0.038013	0.509378	0.64856
128	pww95	9	16	0.081961	0.020106	0.180956	0.2304
129	pww96	12	26	0.281568	0.053093	0.637115	0.8112
130	pww97	10.5	16.2	0.114164	0.020612	0.216426	0.275562
131	pww98	12	25	0.235995	0.049087	0.589049	0.75
132	pww99	11.2	24.5	0.176213	0.047144	0.528007	0.67228
133	pww100	10.5	19.6	0.124619	0.030172	0.316804	0.403368
134	pww101	15.7	26	0.361353	0.053093	0.833559	1.06132
135	pww102	10.4	19.4	0.140351	0.029559	0.307416	0.391414
136	pww103	12	30	0.296515	0.070686	0.84823	1.08
137	pww104	12.6	30.8	0.349919	0.074506	0.938776	1.195286
138	pww105	12.4	33	0.393391	0.08553	1.06057	1.35036
139	pww106	14	32	0.455913	0.080425	1.125947	1.4336
140	pww107	8.2	15	0.046179	0.017671	0.144906	0.1845
141	pww108	15	42.3	0.824048	0.140531	2.107958	2.683935
142	pww109	12.4	21.3	0.154964	0.035633	0.441846	0.562576
143	pww110	10.5	17.3	0.096358	0.023506	0.246815	0.314255
144	pww111	14	21.4	0.23057	0.035968	0.503553	0.641144
145	pww112	9.3	13	0.040927	0.013273	0.123441	0.15717
146	pww113	21	83.6	6.108012	0.548912	11.52714	14.67682
147	pww114	14	50	0.971455	0.19635	2.748894	3.5
148	pww115	20	61	2.202325	0.292247	5.844933	7.442
149	pww116	21.3	54.6	1.588924	0.23414	4.987177	6.349871
150	pww117	17	43.7	1.145585	0.149987	2.549774	3.246473
151	pww118	21.2	53	1.757803	0.220618	4.677109	5.95508
152	pww119	19.3	60.4	1.786667	0.286526	5.529948	7.040949
153	pww120	18.8	51	1.455383	0.204282	3.840503	4.88988
154	pww121	16.3	31.3	0.441257	0.076945	1.254198	1.596895
155	pww122	16	26.3	0.29933	0.054325	0.869203	1.106704
156	pww123	14	22.3	0.201306	0.039057	0.546799	0.696206
157	pww124	15.8	34.5	0.607652	0.093482	1.477016	1.880595
158	pww125	16.6	28.3	0.378011	0.062902	1.044169	1.329477
159	pww126	34	128	15.89807	1.286796	43.75108	55.7056
160	pww127	29.3	94	8.904098	0.693978	20.33355	25.88948
161	pww128	28.5	78.2	6.396678	0.48029	13.68826	17.42843
162	pww129	24	58	2.569473	0.264208	6.340991	8.0736
163	pww130	23.4	46.2	1.456198	0.167639	3.922742	4.99459
164	pww131	16	23	0.265894	0.041548	0.664761	0.8464
165	pww132	25	64.2	3.603007	0.323713	8.092821	10.3041

166	pww133	23.3	47.5	1.831582	0.177205	4.128887	5.257063
167	pww134	22	44.5	1.632743	0.155528	3.421626	4.35655
168	pww135	24.7	62.5	3.411441	0.306796	7.577865	9.648438
169	pww136	15	20	0.189707	0.031416	0.471239	0.6
170	pww137	24.3	65.4	3.671695	0.335927	8.163035	10.3935
171	pww138	26	55	2.788948	0.237583	6.177157	7.865
172	pww139	25.6	78	5.732931	0.477836	12.23261	15.57504
173	pww140	21	59	2.779737	0.273397	5.741339	7.3101
174	pww141	17.4	38.4	0.841345	0.115812	2.015123	2.565734
175	pww142	13	29.2	0.414731	0.066966	0.87056	1.108432
176	pww143	24	84.5	6.059841	0.560794	13.45905	17.1366
177	pww144	16	36	0.741473	0.101788	1.628602	2.0736
178	pww145	22.8	72.5	4.276918	0.412825	9.412408	11.98425
179	pww146	25.8	84.4	6.911923	0.559467	14.43426	18.37827
180	pww147	11.3	17	0.101216	0.022698	0.256487	0.32657
181	pww148	13	19.9	0.18094	0.031103	0.404333	0.514813
182	pww149	17	42	1.125652	0.138544	2.355252	2.9988
183	pww150	11	13.7	0.061415	0.014741	0.162153	0.206459
184	pww151	12	24.2	0.265914	0.045996	0.551953	0.702768
185	pww152	14	23	0.241889	0.041548	0.581666	0.7406
186	pww153	9	18.3	0.104176	0.026302	0.23672	0.301401
187	pww154	14	22	0.199255	0.038013	0.532186	0.6776
188	pww155	16	20.2	0.17615	0.032047	0.512758	0.652864
189	pww156	12.2	23.6	0.175945	0.043744	0.533671	0.679491
190	pww157	10	16	0.074455	0.020106	0.201062	0.256
191	pww158	12	15.6	0.086466	0.019113	0.229361	0.292032
192	pww159	14	24.1	0.248268	0.045617	0.638634	0.813134
193	pww160	11.2	16.5	0.103931	0.021382	0.239484	0.30492
194	pww161	12.2	25	0.235723	0.049087	0.598866	0.7625
195	pww162	13	24.4	0.235089	0.046759	0.607873	0.773968
196	pww163	16	40.7	0.73891	0.1301	2.081607	2.650384
197	pww164	13.6	23	0.242572	0.041548	0.565047	0.71944
198	pww165	13	24.7	0.2279	0.047916	0.622913	0.793117
199	pww166	11.3	25	0.190179	0.049087	0.554687	0.70625
200	pww167	14.2	26.2	0.335669	0.053913	0.765563	0.974745
201	pww168	16.1	27	0.383657	0.057256	0.921814	1.17369
202	pww169	12.4	22.4	0.161215	0.039408	0.488661	0.622182
203	pww170	12	20	0.12753	0.031416	0.376991	0.48
204	pww171	15.2	20.9	0.201818	0.034307	0.521466	0.663951
205	pww172	15	25.8	0.268665	0.052279	0.784189	0.99846
206	pww173	12	16.8	0.109133	0.022167	0.266005	0.338688
207	pww174	15.3	28.9	0.440642	0.065597	1.003638	1.277871

208	pww175	16	27.8	0.415747	0.060699	0.971179	1.236544
209	pww176	17.7	28.5	0.474469	0.063794	1.129153	1.437683
210	pww177	19.5	39.6	0.771023	0.123163	2.401678	3.057912
211	pww178	20.1	30.8	0.615584	0.074506	1.497571	1.906766
212	pww179	23.6	43	1.48927	0.14522	3.427195	4.36364
213	pww180	16.3	24	0.346377	0.045239	0.737395	0.93888
214	pww181	19	31.8	0.750342	0.079423	1.509029	1.921356
215	pww182	12	12.8	0.064122	0.012868	0.154416	0.196608
216	pww183	11.3	14.7	0.055135	0.016972	0.19178	0.244182
217	pww184	10.5	11	0.018133	0.009503	0.099785	0.12705
218	pww185	12	18.9	0.156072	0.028055	0.336662	0.428652
219	pww186	17.5	20.1	0.302507	0.031731	0.55529	0.707018
220	pww187	15.2	20	0.211935	0.031416	0.477522	0.608
221	pww188	16.7	19.3	0.17588	0.029255	0.488563	0.622058
222	pww189	13.2	19.8	0.169681	0.030791	0.406438	0.517493
223	pww190	17.4	23	0.324292	0.041548	0.722928	0.92046
224	pww191	14.4	16	0.111178	0.020106	0.289529	0.36864
225	pww192	12.7	15.3	0.076498	0.018385	0.233494	0.297294
226	pww193	17.4	23.4	0.32618	0.043005	0.748292	0.952754
227	pww194	18	25	0.414664	0.049087	0.883573	1.125
228	pww195	14.3	19.2	0.131714	0.028953	0.414027	0.527155
229	pww196	13	19.4	0.143229	0.029559	0.38427	0.489268
230	pww197	15	22	0.233531	0.038013	0.570199	0.726
231	pww198	13.2	17.1	0.107499	0.022966	0.303149	0.385981
232	pww199	14	19	0.152504	0.028353	0.39694	0.5054
233	pww200	11.2	15.2	0.060178	0.018146	0.203233	0.258765
234	pww201	12	16	0.09261	0.020106	0.241274	0.3072
235	pww202	17.3	31	0.587155	0.075477	1.305748	1.66253
236	pww203	16	25	0.352678	0.049087	0.785398	1
237	pww204	16.3	26.2	0.340898	0.053913	0.87878	1.118897
238	pww205	12.2	16	0.099661	0.020106	0.245296	0.31232
239	pww206	17.3	28.3	0.467506	0.062902	1.0882	1.38554
240	pww207	16.1	30.2	0.570543	0.071631	1.153266	1.468384
241	pww208	20.4	37.3	0.949635	0.109272	2.229142	2.838232
242	pww209	19.3	34.7	0.789986	0.094569	1.825182	2.323894
243	pww210	16.4	31	0.534297	0.075477	1.237819	1.57604
244	pww211	17	24.4	0.394784	0.046759	0.794911	1.012112
245	pww212	20.3	43.4	1.305046	0.147934	3.003069	3.823627
246	pww213	19.4	37	0.90311	0.107521	2.085908	2.65586
247	pww214	19.2	35.9	0.498251	0.101223	1.94348	2.474515
248	pww215	20.5	43.1	1.183106	0.145896	2.990875	3.808101
249	pww216	23.4	54.1	2.108954	0.229871	5.378984	6.848735