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ཚལ་ལྗན་ཁག། རགས་ཚལ་དང་སྤོང་ཀ་ཞབས་
རྟོག་ལས་ཁུངས།



Royal Government of Bhutan
Ministry of Agriculture and Forests
Department of Forests and Park Services

SPECIES SPECIFIC VOLUME EQUATION TO ESTIMATE MERCHANTABLE VOLUME

Pinus roxburghii

Species specific volume equation to
estimate merchantable volume

Pinus roxburghii

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1. Summary

The volume equation developed in this study will predict the merchantable volume of *Pinus roxburghii*. The merchantability standard for volume calculation adopted for this study is 10 cm and above diameter at breast height (dbh) and top diameter measured up to 10 cm over bark.

A total of 16 models were fitted. First 4 models were fitted with volume as a function of diameter at breast height (DBH), while models 5 – 8 were fitted with basal area (BA) as the predictor variable. With product of squared diameter at breast height and height (DBH2H) as predictor variable, 4 models, namely the models 9 – 12 were fitted. The last four models, 13 -16 were fitted with product of basal area and height (BAH) as the predictor.

The initial plots of response variable (volume) and predictor variables (DBH, BA, DBH2H and BAH) clearly indicated presence of heteroscedasticity, which has been modeled using variance functions (varFixed, varPower and varConstPower) in gls () function of nlme package.

Of the sixteen, two models viz model 7(fitted without height as predictor) and model 15 (fitted with height as predictor) with lowest values of AIC and BIC have been selected as the best fit models for *Pinus roxburghii*. The model 7 had AIC and BIC values of 9 and 17 respectively, while the model 15 had AIC and BIC values of 11 and 19 respectively. Lower the AIC and BIC values, better the fit of the model.

The performance of the selected models was assessed by comparing the actual volume with the volumes predicted by two selected models for each tree. From the assessment, we observed that the model 15 which uses height outperforms the model 7.

2. Introduction

The volume equations, developed during pre-investment survey (PIS) carried out between 1974-8 predict total tree volume, and not the predict merchantable volume of trees. The recent change of policy of the Department of Forests and Park Services to allot timber for rural house construction in the form of log volume instead of allotting by number of trees as was once practiced, has necessitated development of merchantable log volume equation.

Therefore, standards of merchantability adopted for this study to develop merchantable log volume equation are trees of 10 cm and above diameter at breast height (dbh) and the sections up to 10 cm top diameter over the bark.

As was done for PIS exercise to develop volume equation, this study ignores/does not consider the volume of foliage and branches for the purpose of calculating the merchantable volume. This decision stems from the objective, which is to estimate merchantable log volume. Moreover, branches are rarely used as timber (at least in Bhutan) and are mostly used for firewood.

The sample trees for this study have been felled as part of biomass equation development field work. The data protocol for biomass equation development required collecting a minimum of 8 trees each from four regions of Bhutan namely, eastern, eastern central, western and western central. However, 40 trees in total have been felled for *Pinus roxburghii* from three regions namely; eastern, east-central and western-central regions.

The trees were felled at 0.3 m height from the ground at which the diameter was measured and recorded. After felling diameter was measured at 0.7 m from 0.3 m height (essentially making 1 m height, i.e. $0.3\text{ m} + 0.7\text{ m} = 1\text{ m}$). Thereafter, at every meter length, the diameter was measured and recorded, thus making many 1 m length sections of log. As mentioned above the smallest top diameter considered for merchantable log volume calculation was up to 10 cm diameter over bark. Top sections below 10 cm diameter have been discarded.

3. Volume Calculation

Trees after felling are converted into different sizes of sections depending on the requirement and demand. Sections with length of 8 or more feet long are called logs and shorter ones are called sticks or bolts (Avery and Burkhardt, 1994). The scaling or measuring the volume of the section is done by multiplying the length with the cross-sectional area of the section. Although they rarely form true circles, they are assumed so for the purpose of calculating cross sectional area in meter square, which is;

$$\text{Cross sectional area (A)} = A = \pi r^2 = \frac{\pi D^2}{4 \times 10000} \quad (1)$$

Where **r** is radius in meters and **D** is diameter at breast height in centimeters.

From the ground level to 0.3 m height (height at which sample tree has been cut) is section I, while 0.3 m to 0.7 m is section II. The subsequent sections of 1 m length each are numbered III, IV and so on. The last section is the terminal section, whose length is equal to or less than 1 m. As was adopted for PIS, in this study too the branch volumes are ignored assuming that rarely branches yield merchantable timber.

The diameter at zero height (ground level) for stump wasn't measured in the field (for those sample trees for which volume data was collected during biomass equation development field work) and therefore, calculated based on diameter reading at 0.3 m height. Therefore, diameter at zero height was calculated as 10% more than diameter at 0.3 m height, which is;

$$D_{(\text{ground})} = D_{(0.3 \text{ m})} + 10\% * D_{(0.3 \text{ m})} \quad (2)$$

Where;

$D_{(\text{ground})}$ is diameter in centimeter of tree at ground level
 $D_{(0.3 \text{ m})}$ is diameter in centimeter of tree at 0.3 m height

For instance, if $D_{(0.3 \text{ m})}$ was 70 cm, the $D_{(\text{ground})}$ is calculated as;

$$\begin{aligned} D_{(\text{ground})} &= 70 \text{ cm} + 10\% \text{ of } 70 \text{ cm} \\ &= 70 + 7 \\ &= 77 \text{ cm} \end{aligned}$$

The most commonly used formulae for calculating volume are the Huber, Newton and Smalian's formulae (Sadiq, 2006, and Goulding, 1979). Of the three commonly used volume calculation approaches or formulae, the Smalian's formula has been used to calculate volume (in m³) for this study, as under;

$$\text{Section volume (V}_s) = \frac{A+a}{2} * L \quad (3)$$

Where **A** = Cross sectional area in m² at large end of the section
a = Cross sectional area in m² at small end of the section
L = Length of the section in meter

Smalian's formula is the easiest and least expensive to apply and therefore applied to get volume for each section of the sample trees. However, for the terminal section, the following formula was used to calculate the volume;

$$\text{Terminal section volume (V}_t) = \frac{A}{3} * L \quad (4)$$

The volume for sections and terminal section for individual trees were then summed to obtain the total volume for each individual sample tree, which is;

$$\text{Volume of tree (V)} = \sum_{s=1}^n V_s + V_t \quad (5)$$

After obtaining individual tree volume (Volume.m3), it was then tabulated against the variables - height in meter (Height.m) and the diameter at breast height in centimeter (DBH.cm).

4. The Dataset used for modeling volume of *Pinus roxburghii*

A total of 40 trees have been felled and collected data for developing volume equations for *Pinus roxburghii* from 3 regions, namely eastern, eastern central and western central. Summary of the dataset is presented below, while the detailed one is provided as an annexure.

4.1 Summary descriptive statistics of *Pinus roxburghii* dataset

> summary(pr)

Tree_ID	Height.m	DBH.cm	Volume.m3
pre01 : 1	Min. :10.85	Min. :14.10	Min. :0.08249
pre02 : 1	1st Qu.:22.35	1st Qu.:28.95	1st Qu.:0.79571
pre04 : 1	Median :27.25	Median :46.00	Median :2.40590
pre05 : 1	Mean :27.70	Mean :46.00	Mean :2.90982
pre06 : 1	3rd Qu.:35.29	3rd Qu.:62.60	3rd Qu.:5.17945
pre07 : 1	Max. :41.90	Max. :77.80	Max. :8.27103

BA.m2	BAH.m3	DBH2H.m3
Min. :0.01561	Min. : 0.1694	Min. : 0.2157
1st Qu.:0.06583	1st Qu.: 1.5702	1st Qu.: 1.9992
Median :0.16619	Median : 4.5409	Median : 5.7816
Mean :0.19497	Mean : 6.3287	Mean : 8.0580
3rd Qu.:0.30786	3rd Qu.:11.2558	3rd Qu.:14.3314
Max. :0.47539	Max. :17.8128	Max. :22.6800

5. Fitting the models

The models have been fitted in R, which is a robust statistical computing environment. It is a powerful tool which provides wide range of statistical and graphical options to explore, calculate and manage data besides modelling. It is very powerful and widely used statistical tool which is free and allows user to customize the scripts depending on desired output, which is not possible in many of the statistical softwares.

After reading in the excel files into R, we created other variables namely; basal area in square meter (BA.m2), basal area in meter times height in meter (BAH.m3) and square of the diameter in meter times height in meter (DBH2H.m3). The height in meter (Height.m) and diameter in centimeter (DBH.cm) were measured and recorded in the field.

Prior to fitting models, we explored and examined each set of data by preparing descriptive summaries that provided mean, median and range of dependent/response and independent/predictor variables. Then we plotted scatter graphs which provided sense of relationship between the dependent/response (volume) and independent/predictor variables (namely DBH.cm, BA.m2, DBH2H.m3 and BAH.m3). These graphs showed curvilinear relationship between response and predictor variables. The scatter plots also clearly revealed the presence of phenomenon, referred in statistical parlance, as heteroscedasticity, which is the increase in variation in response (volume) variable with increase in value of the predictor variables.

Therefore, we fitted the models using the `gls ()` function of the `nlme` package of R, because the `gls ()` function has the capability to model heteroscedasticity. We didn't transform the variables, mainly response variable, because transformation makes it difficult to directly interpret the relationship between response and predictor variables; and secondly to compare the AIC and BIC values among the different models, the response variables need to be identical.

The models were fitted with volume as a function of four variables;

- 1) DBH.cm,
- 2) BA.m2,
- 3) DBH2H.m3 and
- 4) BAH.m3.

For each of the variable, we fitted one simple `gls ()` function, which can be written in the following form;

$$Y = \beta_0 + \beta_1 X + \epsilon, \quad (6)$$

Where $Y = \text{Volume (V)}$ and $X = \text{predictor variable}$

And then fitted 3 models with restricted natural cubic spline functions. The restricted natural cubic spline function enables better tracking of curvilinear relationship between response and predictor variables. These models introduce an additional predictor variable as part of a 3 knot-cubic spline. They take the following forms;

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \epsilon, \quad (7)$$

Where $Y = \text{Response variable, volume (V)}$

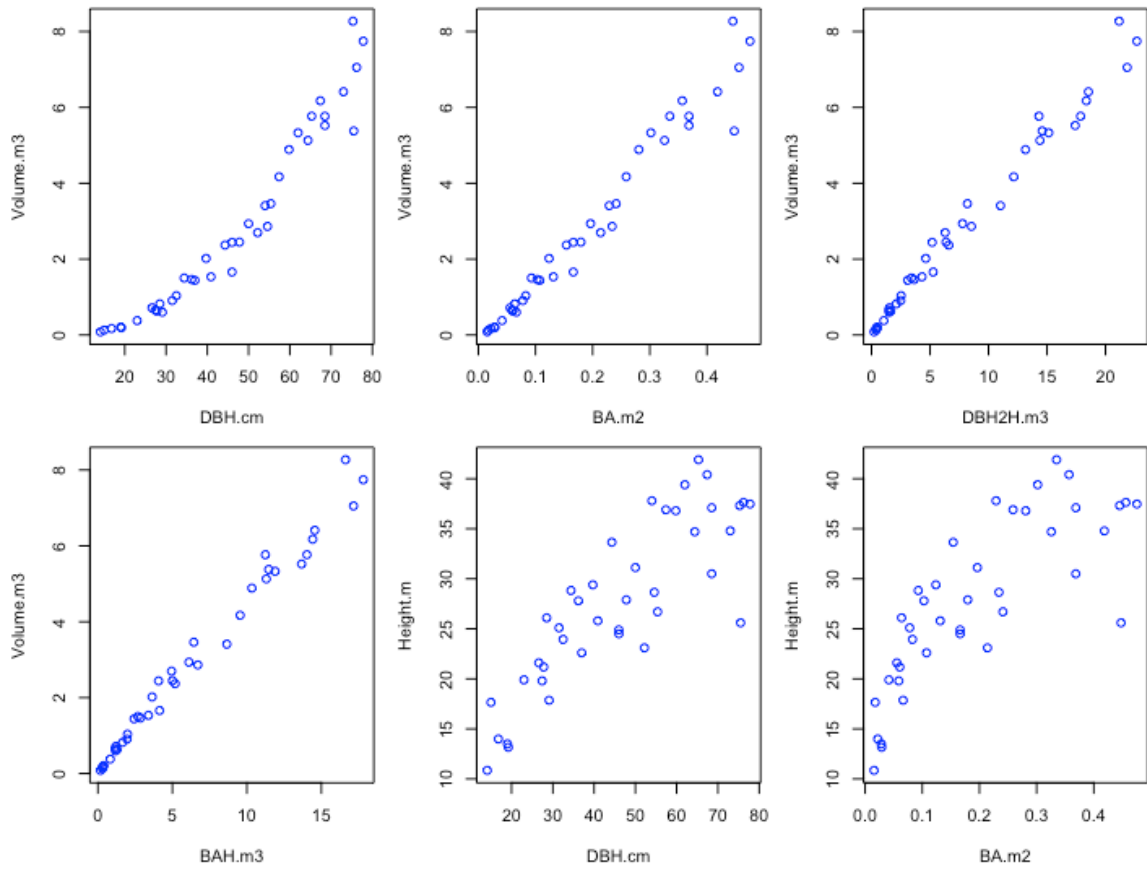
$X_1 = \text{Predictor variable}$

$X_2 = g(X_1)$

And $g(X_1)$ is the spline transformation of X_1 predictor variable

6. Summary Plots

Pinus roxburghii (N = 40)



7. Models and results

7.1 Model 1 - Volume with diameter at breast height (DBH) as predictor

```
> pr.m1 <- gls(Volume.m3 ~ DBH.cm)
> summary(pr.m1)
```

Generalized least squares fit by REML

Model: Volume.m3 ~ DBH.cm

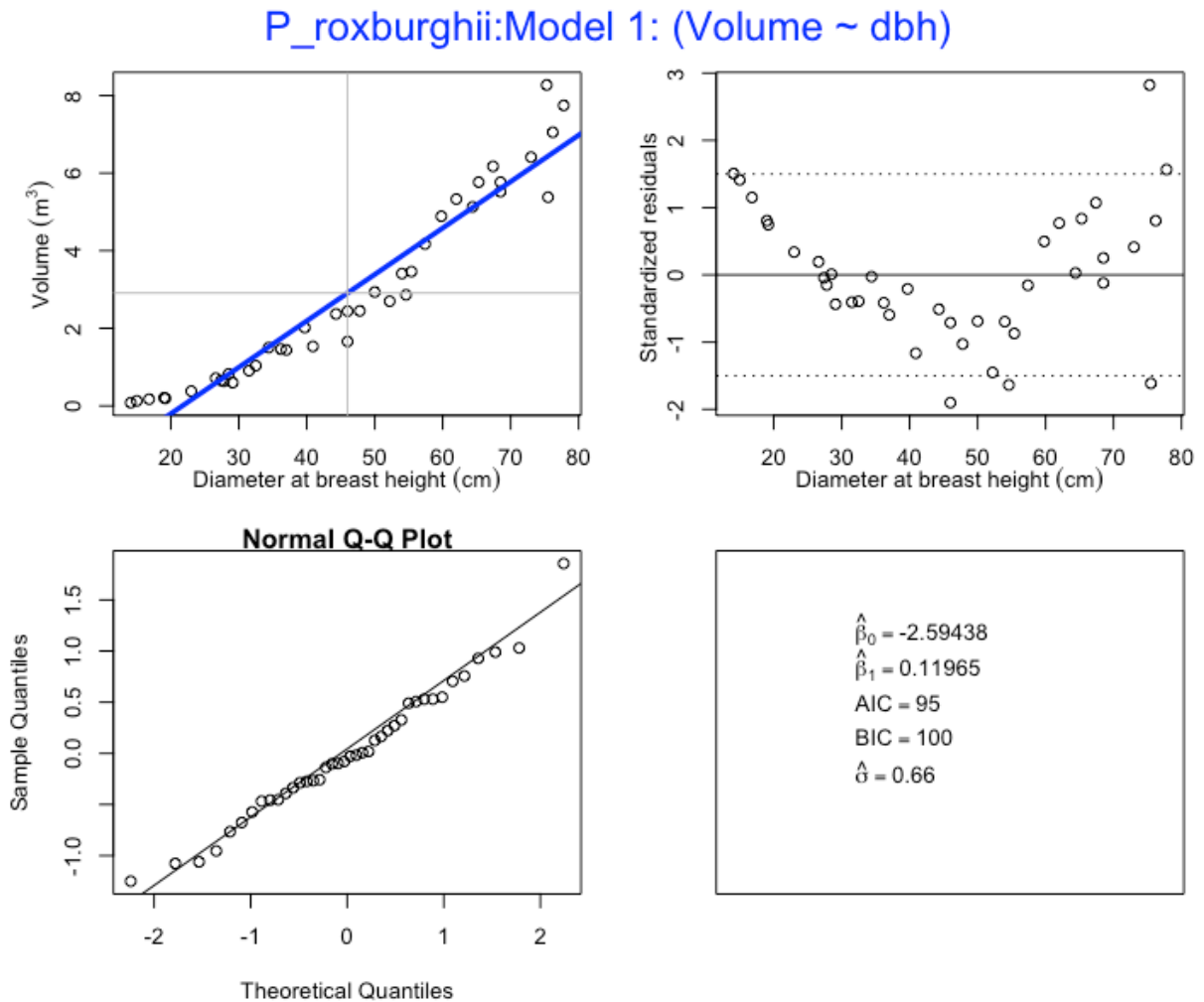
Data: NULL

	AIC	BIC	logLik
	95.21764	100.1304	-44.60882

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	-2.594379	0.27054406	-9.589487	0
DBH.cm	0.119650	0.00542998	22.035072	0

Plot of model 1



7.2 Model 2 - Volume with diameter at breast height (DBH) as predictor, with varFixed

```
> pr.m2 <- gls(Volume.m3 ~ DBH.cm + DBH.cm.splinepoints,
               na.action=na.omit, weights = varFixed(~DBH.cm))
> summary(pr.m2)
```

Generalized least squares fit by REML

Model: Volume.m3 ~ DBH.cm + DBH.cm.splinepoints

Data: NULL

	AIC	BIC	logLik
	74.97101	81.41468	-33.4855

Variance function:

Structure: fixed weights

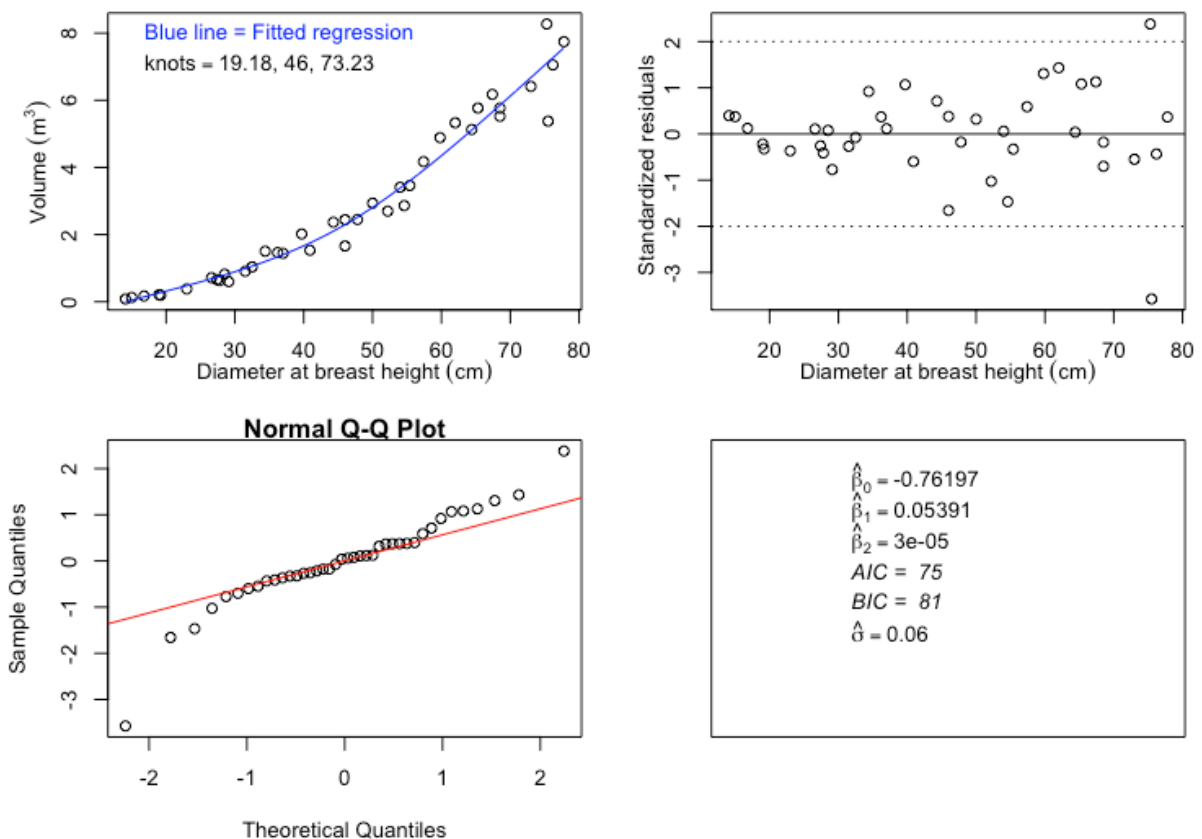
Formula: ~DBH.cm

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	-0.7619669	0.19313498	-3.945256	3e-04
DBH.cm	0.0539127	0.00692077	7.789981	0e+00
DBH.cm.splinepoints	0.0000300	0.00000345	8.688129	0e+00

Plot of Model 2

P_roxburghii:Model 2 : (Volume ~ dbh), Cubic spline with varFixed



7.3 Model 3- Volume with diameter at breast height (DBH) as predictor, with varPower

```
> pr.m3 <- gls(Volume.m3 ~ DBH.cm + DBH.cm.splinepoints,
  na.action=na.omit, weights = varPower(form =
  ~DBH.cm))
> summary(pr.m3)
```

Generalized least squares fit by REML

Model: Volume.m3 ~ DBH.cm + DBH.cm.splinepoints

Data: NULL

	AIC	BIC	logLik
	56.10397	64.15856	-23.05198

Variance function:

Structure: Power of variance covariate

Formula: ~DBH.cm

Parameter estimates:

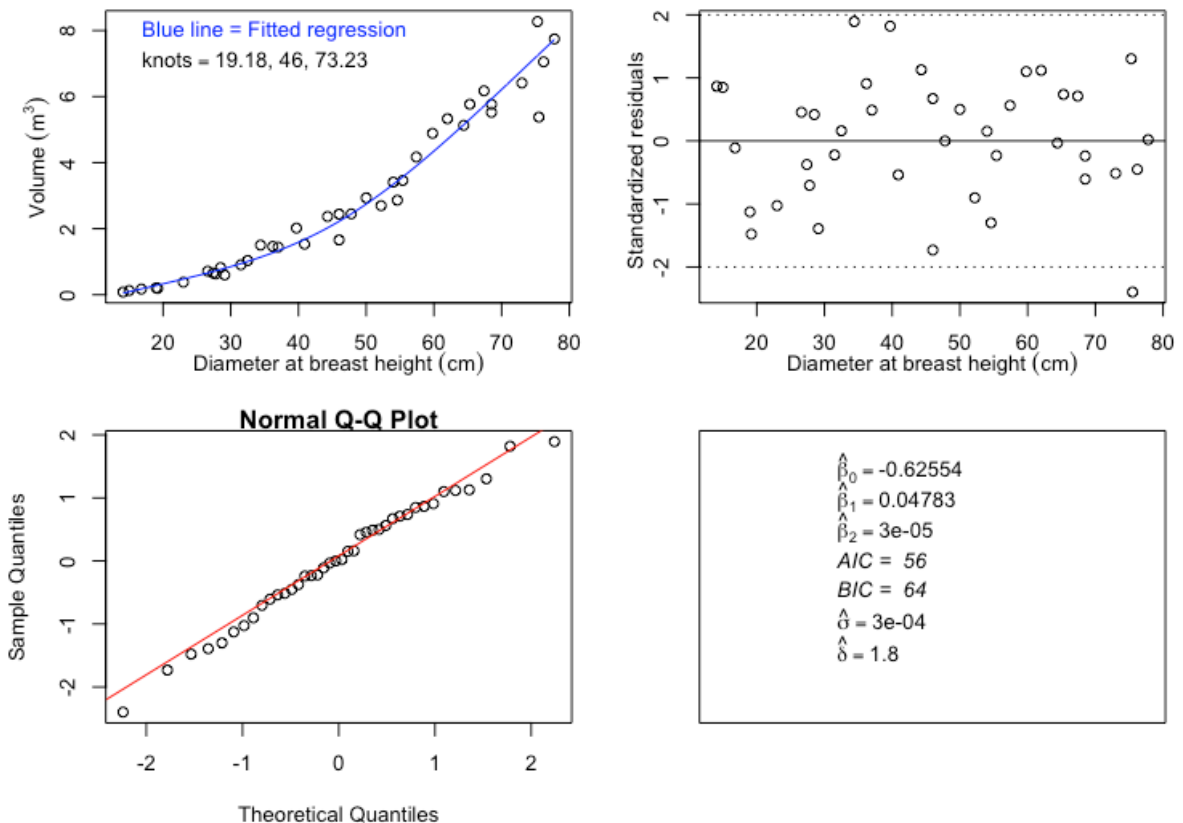
power
1.798497

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	-0.6255356	0.06410160	-9.758501	0
DBH.cm	0.0478313	0.00323943	14.765344	0
DBH.cm.splinepoints	0.0000336	0.00000271	12.425500	0

Plot of Model 3

P_roxburghii:Model 3: (Volume ~ dbh), Cubic spline with varPower



7.4 Model 4- Volume with diameter at breast height (DBH) as predictor, with varConstPower

```
> pr.m4 <- gls(Volume.m3 ~ DBH.cm + DBH.cm.splinepoints,
               na.action=na.omit, weights = varConstPower(form =
               ~DBH.cm))
> summary(pr.m4)
```

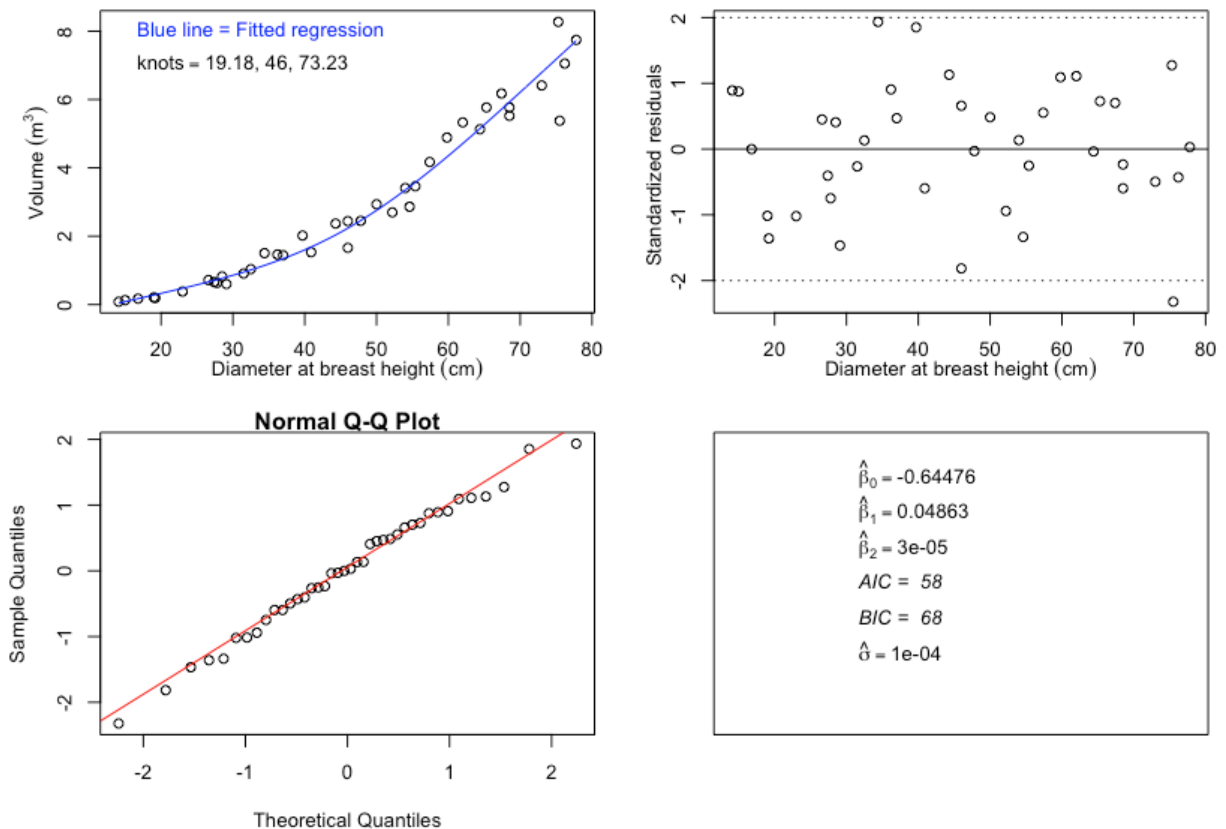
```
Generalized least squares fit by REML
Model: Volume.m3 ~ DBH.cm + DBH.cm.splinepoints
Data: NULL
      AIC      BIC    logLik
58.03329 67.6988 -23.01665
```

```
Variance function:
Structure: Constant plus power of variance covariate
Formula: ~DBH.cm
Parameter estimates:
      const      power
125.488182   1.990168
```

```
Coefficients:
              Value Std.Error  t-value p-value
(Intercept)  -0.6447622 0.06888792  -9.359583      0
DBH.cm        0.0486286 0.00331757  14.657924      0
DBH.cm.splinepoints 0.0000333 0.00000275 12.093231      0
```

Plot of Model 4

P_roxburghii:Model 4: (Volume ~ dbh), Cubic spline with varConstPower



7.5 Model 5 - Volume with basal area (BA) as predictor

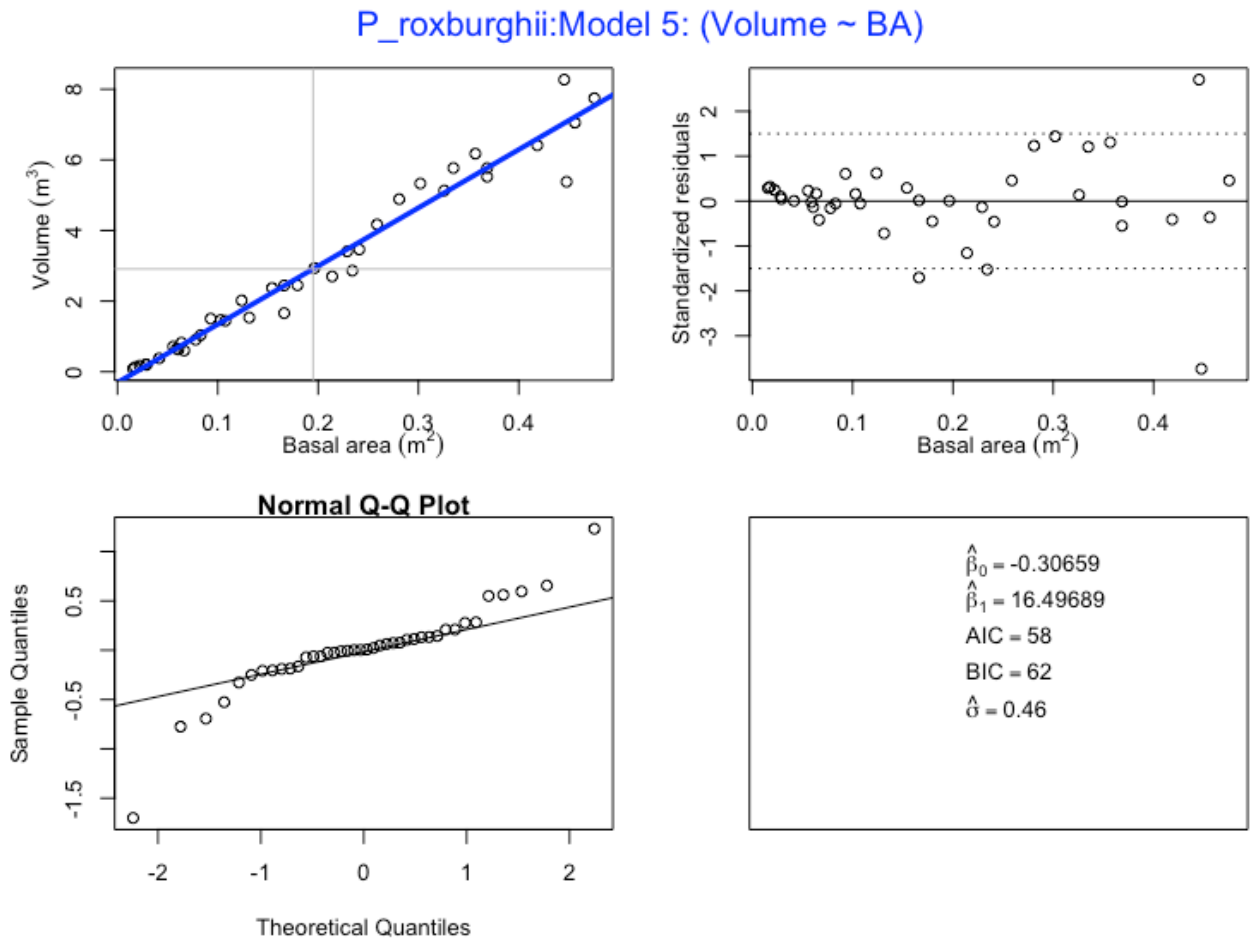
```
> pr.m5 <- gls(Volume.m3 ~ BA.m2)
> summary(pr.m5)
```

Generalized least squares fit by REML

```
Model: Volume.m3 ~ BA.m2
Data: NULL
      AIC      BIC    logLik
57.50613 62.41889 -25.75306
```

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	-0.306588	0.1225028	-2.50270	0.0167
BA.m2	16.496892	0.5083931	32.44909	0.0000

Plot of Model 5

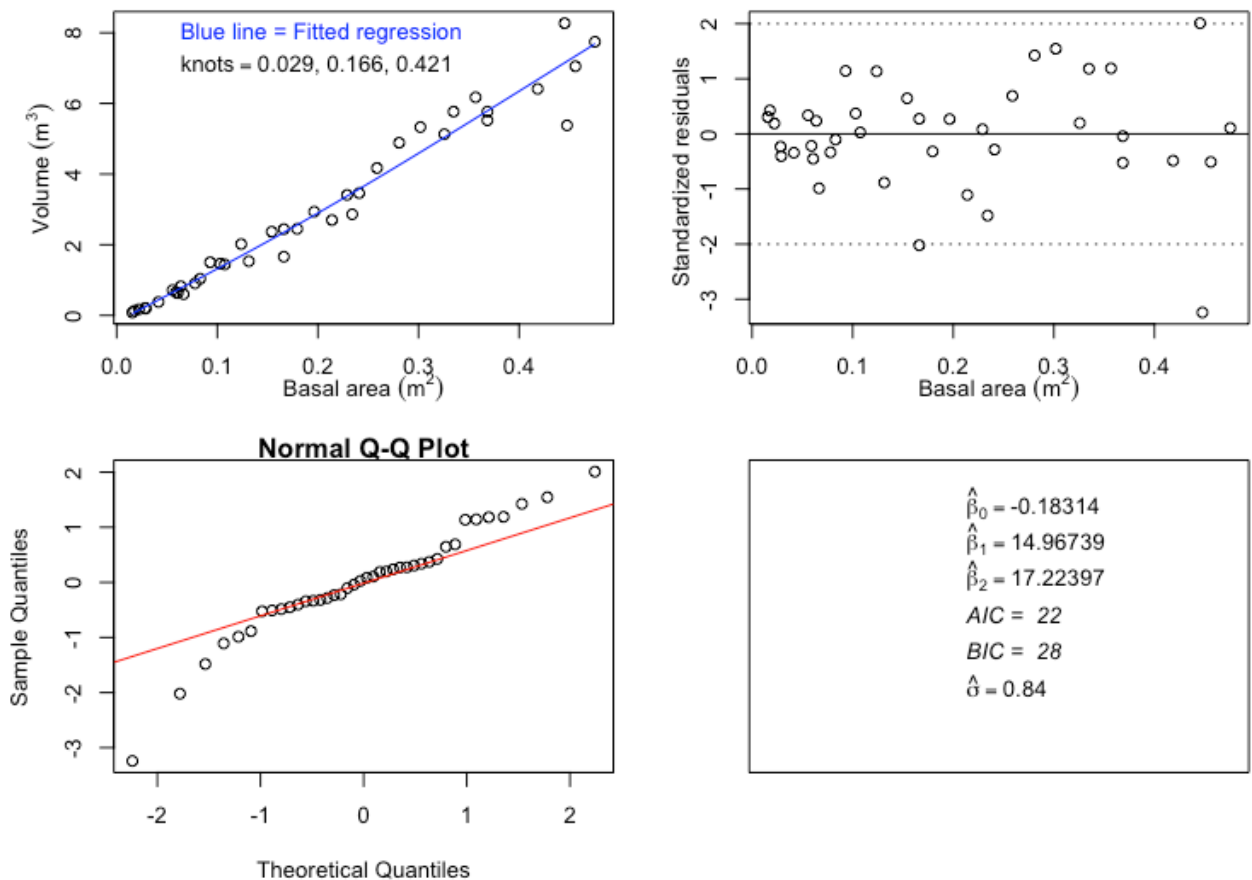
7.6 Model 6 - Volume with basal area (BA) as predictor, with varFixed

```
> pr.m6<- gls(Volume.m3 ~ BA.m2 + BA.m2.splinepoints,
              na.action=na.omit, weights = varFixed(~BA.m2))
> summary(pr.m6)
```

```
Generalized least squares fit by REML
Model: Volume.m3 ~ BA.m2 + BA.m2.splinepoints
Data: NULL
      AIC      BIC    logLik
22.04746 28.49113 -7.02373
```

```
Variance function:
Structure: fixed weights
Formula: ~BA.m2
```

```
Coefficients:
                Value Std.Error   t-value p-value
(Intercept)    -0.183136  0.062263  -2.941325  0.0056
BA.m2           14.967393  0.918798  16.290189  0.0000
BA.m2.splinepoints 17.223965 12.203854   1.411355  0.1665
```

Plot of Model 6**P_roxburghii:Model 6: (Volume ~ BA), Cubic spline with varFixed**

7.7 Model 7 Volume with basal area (BA) as predictor, with varPower

```
> pr.m7 <- gls(Volume.m3 ~ BA.m2 + BA.m2.splinepoints,
               na.action=na.omit, weights = varPower(form = ~BA.m2))
> summary(pr.m7)
```

Generalized least squares fit by REML

Model: Volume.m3 ~ BA.m2 + BA.m2.splinepoints

Data: NULL

AIC	BIC	logLik
9.035148	17.08974	0.4824258

Variance function:

Structure: Power of variance covariate

Formula: ~BA.m2

Parameter estimates:

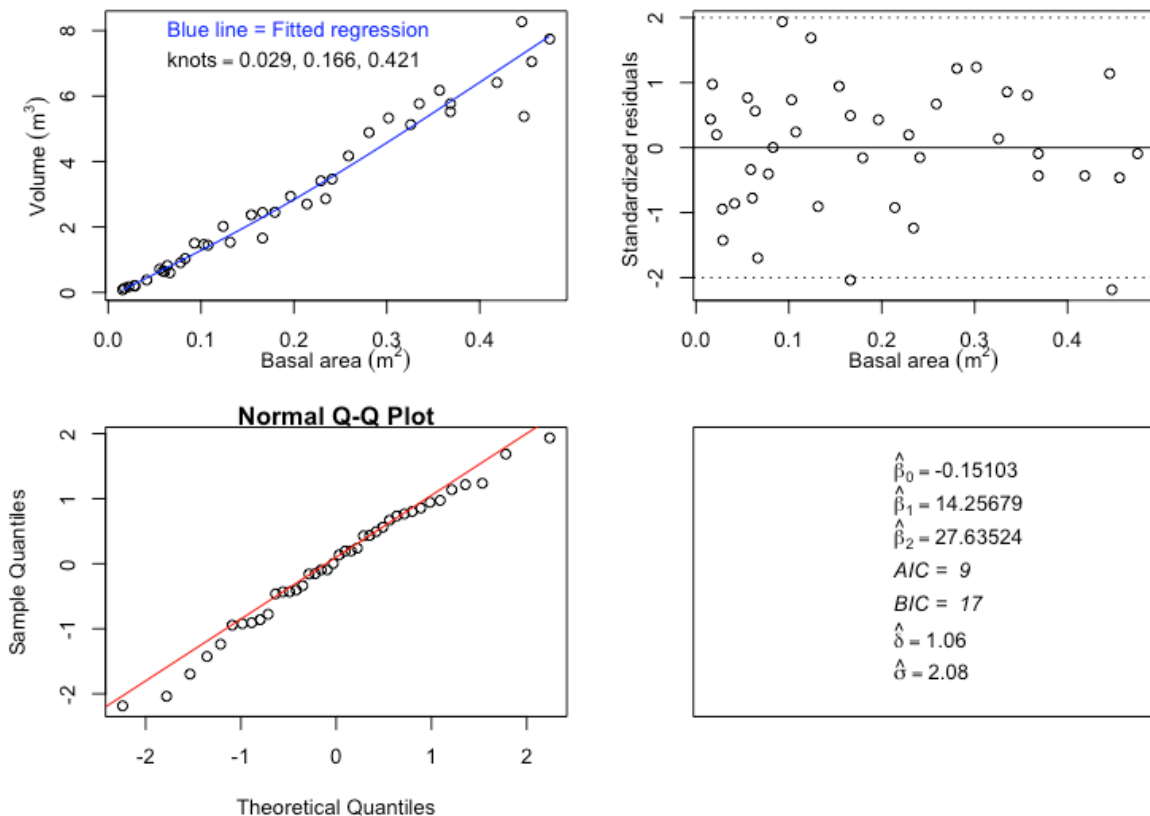
power
1.062215

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	-0.151034	0.021977	-6.872275	0.0000
BA.m2	14.256793	0.609993	23.372054	0.0000
BA.m2.splinepoints	27.635236	11.405877	2.422894	0.0204

Plot of Model 7

P_roxburghii:Model 7: (Volume ~ BA), Cubic spline with varPower



7.8 Model 8 – Volume with basal area (BA) as predictor, with varConstPower

```
> pr.m8 <- gls(Volume.m3 ~ BA.m2 + BA.m2.splinepoints,
  na.action=na.omit, weights = varConstPower(form =
  ~BA.m2))
> summary(pr.m8)
```

Generalized least squares fit by REML

Model: Volume.m3 ~ BA.m2 + BA.m2.splinepoints

Data: NULL

AIC	BIC	logLik
11.03515	20.70066	0.4824258

Variance function:

Structure: Constant plus power of variance covariate

Formula: ~BA.m2

Parameter estimates:

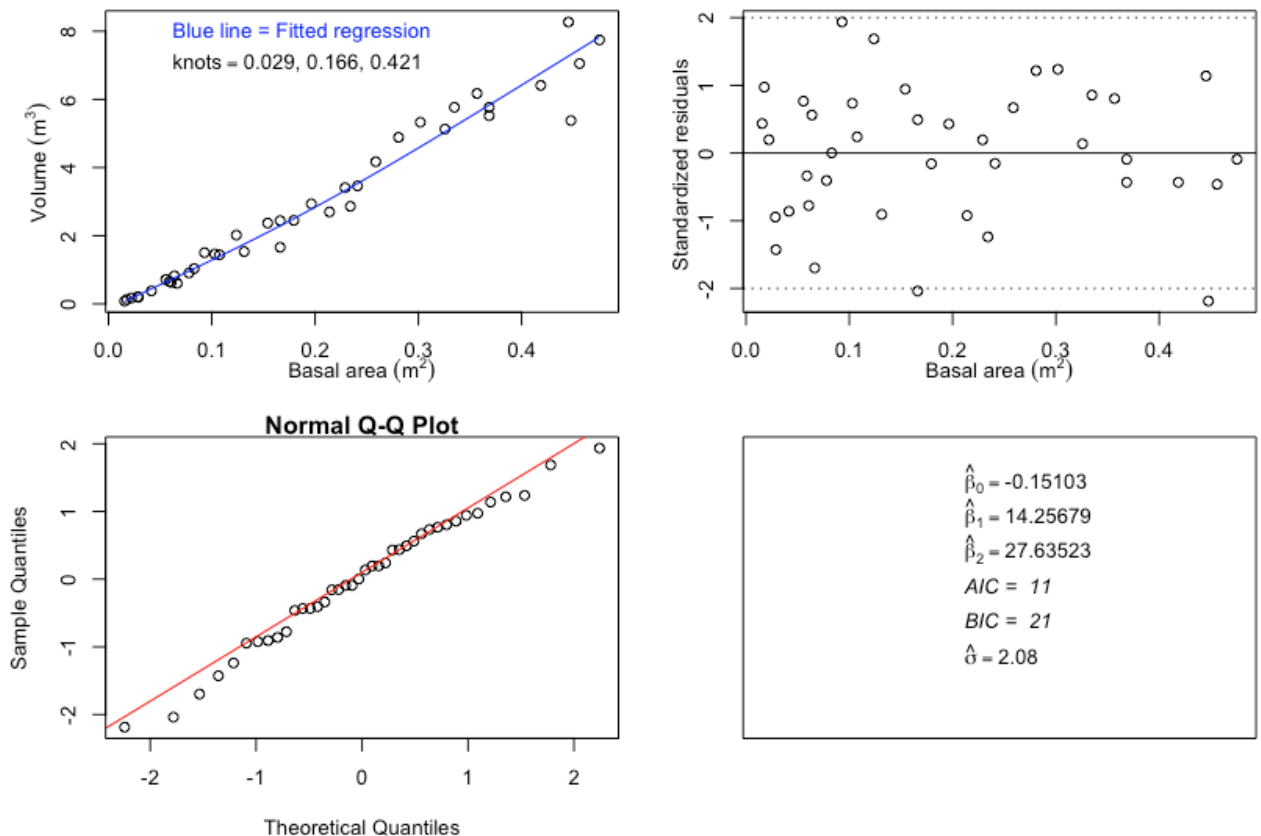
const	power
6.713215e-11	1.062215e+00

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	-0.151034	0.021977	-6.872273	0.0000
BA.m2	14.256793	0.609993	23.372052	0.0000
BA.m2.splinepoints	27.635231	11.405876	2.422894	0.0204

Plot of Model 8

P_roxburghii:Model 8: (Volume ~ BA), Cubic spline with varConstPower



7.9 Model 9 – Volume with square of diameter at breast height * height (DBH2H) as predictor

```
> pr.m9 <- gls(Volume.m3 ~ DBH2H.m3)
> summary(pr.m9)
```

Generalized least squares fit by REML

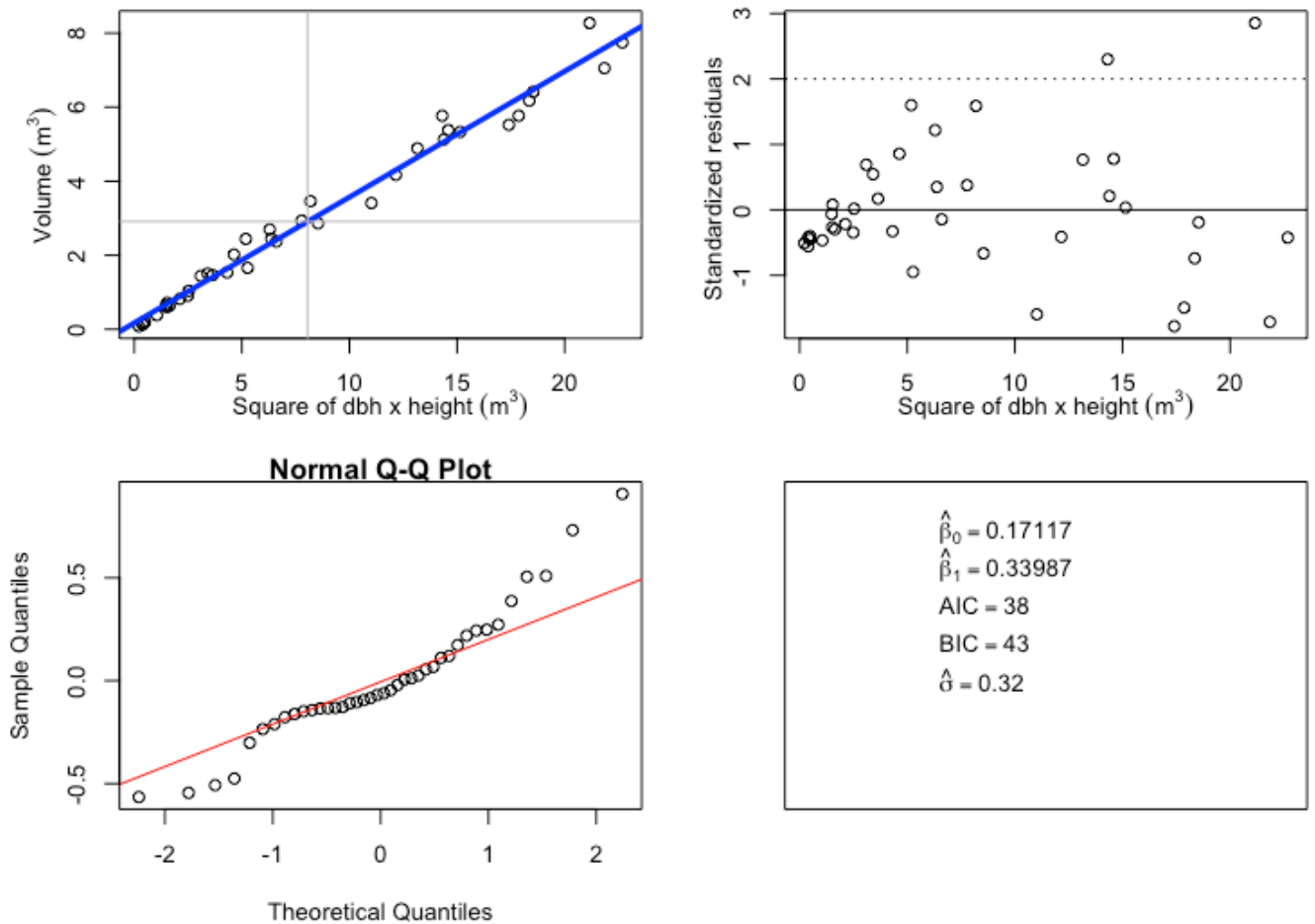
```
Model: Volume.m3 ~ DBH2H.m3
Data: NULL
      AIC      BIC    logLik
38.03745 42.95021 -16.01872
```

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	0.1711651	0.07709398	2.22021	0.0324
DBH2H.m3	0.3398695	0.00725116	46.87102	0.0000

Plot of Model 9

P_roxburghii:Model 9: (Volume ~ dbh^2*H)



7.10 Model 10 – Volume with square of diameter at breast height * height (DBH2H) as predictor, with varFixed

```
> pr.m10 <- gls(Volume.m3 ~ DBH2H.m3 + DBH2H.m3.splinepoints,
                na.action=na.omit, weights = varFixed(~DBH2H.m3))
> summary(pr.m10)
```

Generalized least squares fit by REML

```
Model: Volume.m3 ~ DBH2H.m3 + DBH2H.m3.splinepoints
Data: NULL
      AIC      BIC    logLik
21.34317 27.78684 -6.671586
```

Variance function:

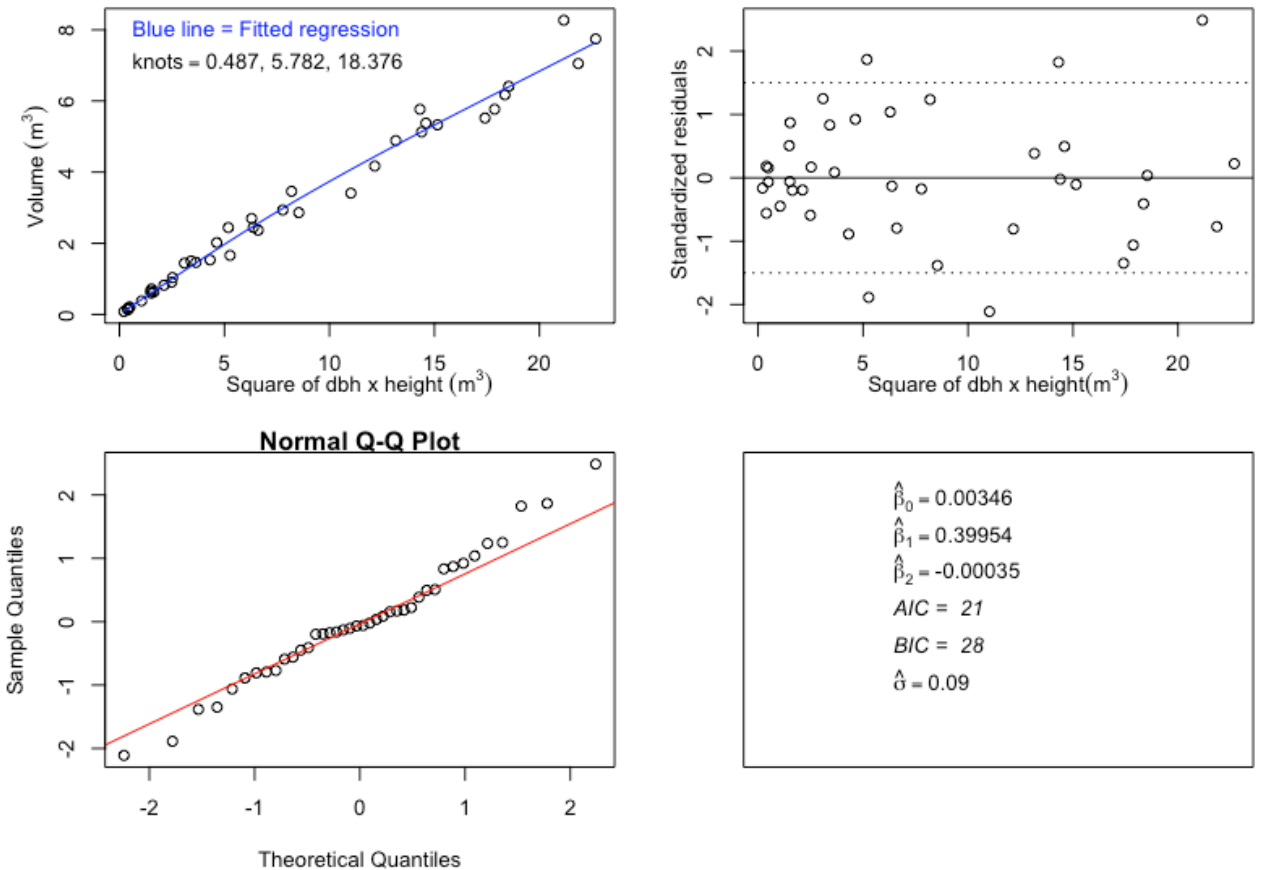
```
Structure: fixed weights
Formula: ~DBH2H.m3
```

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	0.0034553	0.026436113	0.130705	0.8967
DBH2H.m3	0.3995445	0.014504636	27.545990	0.0000
DBH2H.m3.splinepoints	-0.0003458	0.000103002	-3.357631	0.0018

Plot of Model 10

P_roxburghii:Model 10: (Volume ~ dbh^2*H), Cubic Spline with varFixed



7.11 Model 11– Volume with square of diameter at breast height * height (DBH2H) as predictor, with varPower

```
> pr.m11 <- gls(Volume.m3 ~ DBH2H.m3 + DBH2H.m3.splinepoints,
  na.action=na.omit, weights = varPower(form =
  ~DBH2H.m3))
> summary(pr.m11)
```

Generalized least squares fit by REML

```
Model: Volume.m3 ~ DBH2H.m3 + DBH2H.m3.splinepoints
Data: NULL
      AIC      BIC    logLik
13.20905 21.26364 -1.604523
```

Variance function:

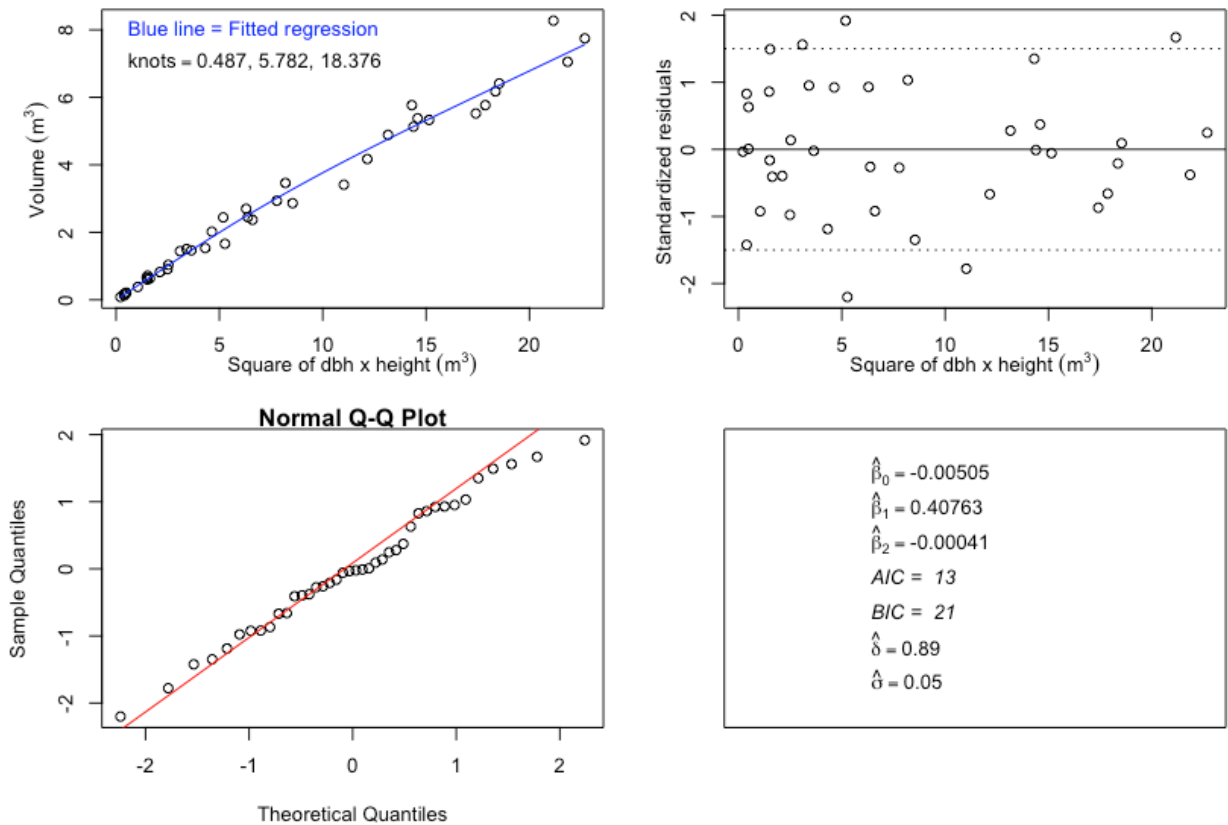
```
Structure: Power of variance covariate
Formula: ~DBH2H.m3
Parameter estimates:
  power
0.8935963
```

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	-0.0050467	0.009373344	-0.53841	0.5935
DBH2H.m3	0.4076275	0.011226478	36.30947	0.0000
DBH2H.m3.splinepoints	-0.0004080	0.000102107	-3.99543	0.0003

Plot of Model 11

P_roxburghii:Model 11: (Volume ~ dbh^2*H), Cubic Spline with varPower



7.12 Model 12 –Volume with square of diameter at breast height * height (DBH2H) as predictor, with varConstPower

```
> pr.m12 <- gls(Volume.m3 ~ DBH2H.m3 + DBH2H.m3.splinepoints,
  na.action=na.omit, weights = varConstPower(form =
    ~DBH2H.m3))
> summary(pr.m12)
```

Generalized least squares fit by REML

```
Model: Volume.m3 ~ DBH2H.m3 + DBH2H.m3.splinepoints
Data: NULL
      AIC      BIC    logLik
15.20905 24.87455 -1.604523
```

Variance function:

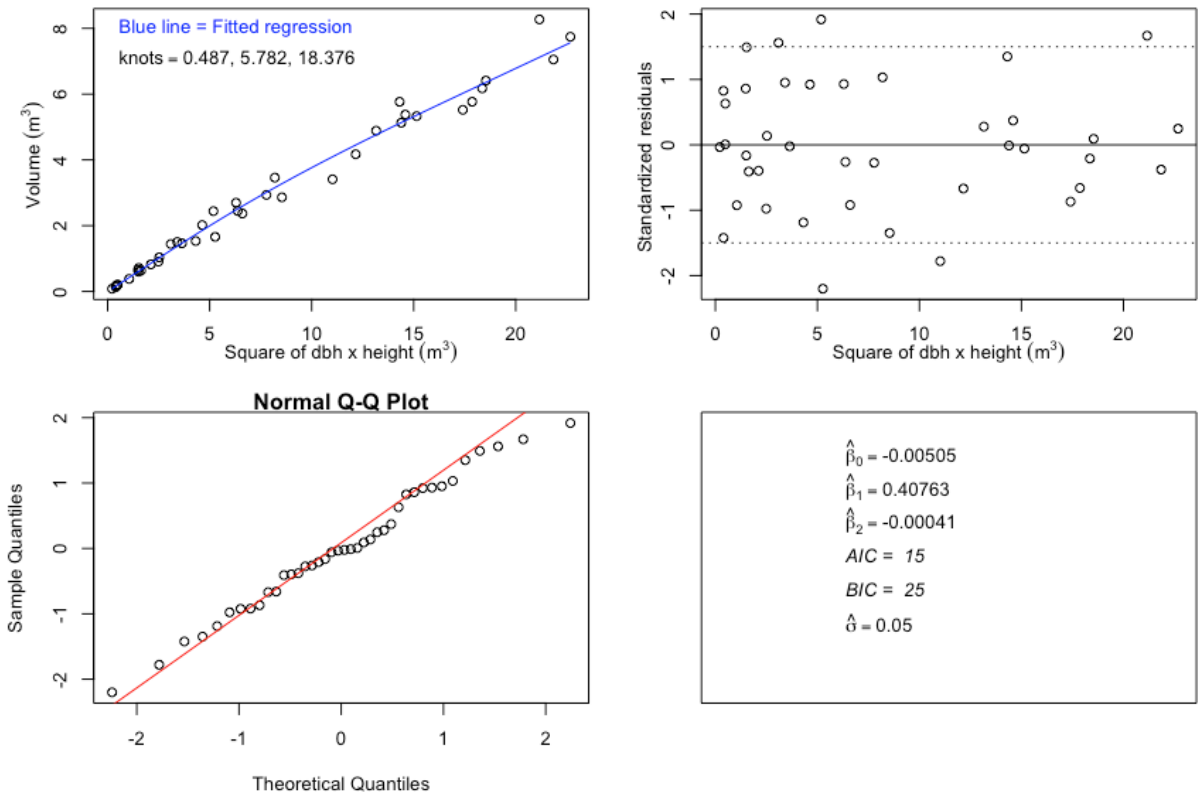
```
Structure: Constant plus power of variance covariate
Formula: ~DBH2H.m3
Parameter estimates:
      const      power
6.110951e-10 8.935965e-01
```

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	-0.0050467	0.009373340	-0.53841	0.5935
DBH2H.m3	0.4076275	0.011226477	36.30947	0.0000
DBH2H.m3.splinepoints	-0.0004080	0.000102107	-3.99543	0.0003

Plot of Model 12

P_roxburghii:Model 12: (Volume ~ dbh^2*H), Cubic Spline with varConstPower



7.13 Model 13 – Volume with basal area * height (BAH) as predictor

```
> pr.m13 <- gls(Volume.m3 ~ BAH.m3)
> summary(pr.m13)
```

Generalized least squares fit by REML

Model: Volume.m3 ~ BAH.m3

Data: NULL

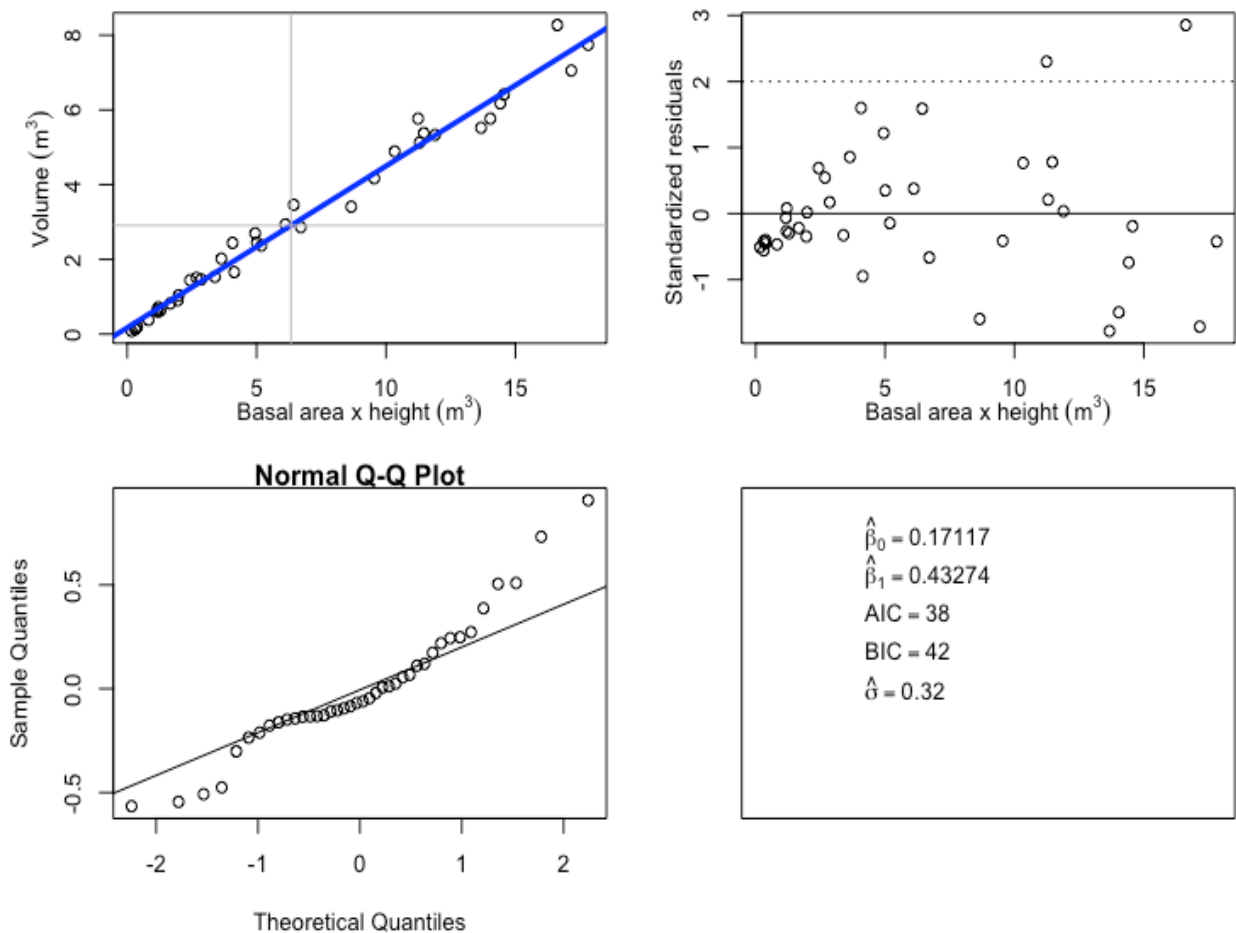
	AIC	BIC	logLik
	37.55432	42.46708	-15.77716

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	0.1711651	0.07709398	2.22021	0.0324
BAH.m3	0.4327352	0.00923247	46.87102	0.0000

Plot of Model 13

P_roxburghii:Model 13: (Volume ~ BAH)



7.14 Model 14 – Volume with basal area * height (BAH) as predictor, with varFixed

```
> pr.m14 <- gls(Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints,
                na.action=na.omit, weights = varFixed(~BAH.m3))
> summary(pr.m14)
```

Generalized least squares fit by REML
 Model: Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints
 Data: NULL
 AIC BIC logLik
 19.41066 25.85433 -5.705328

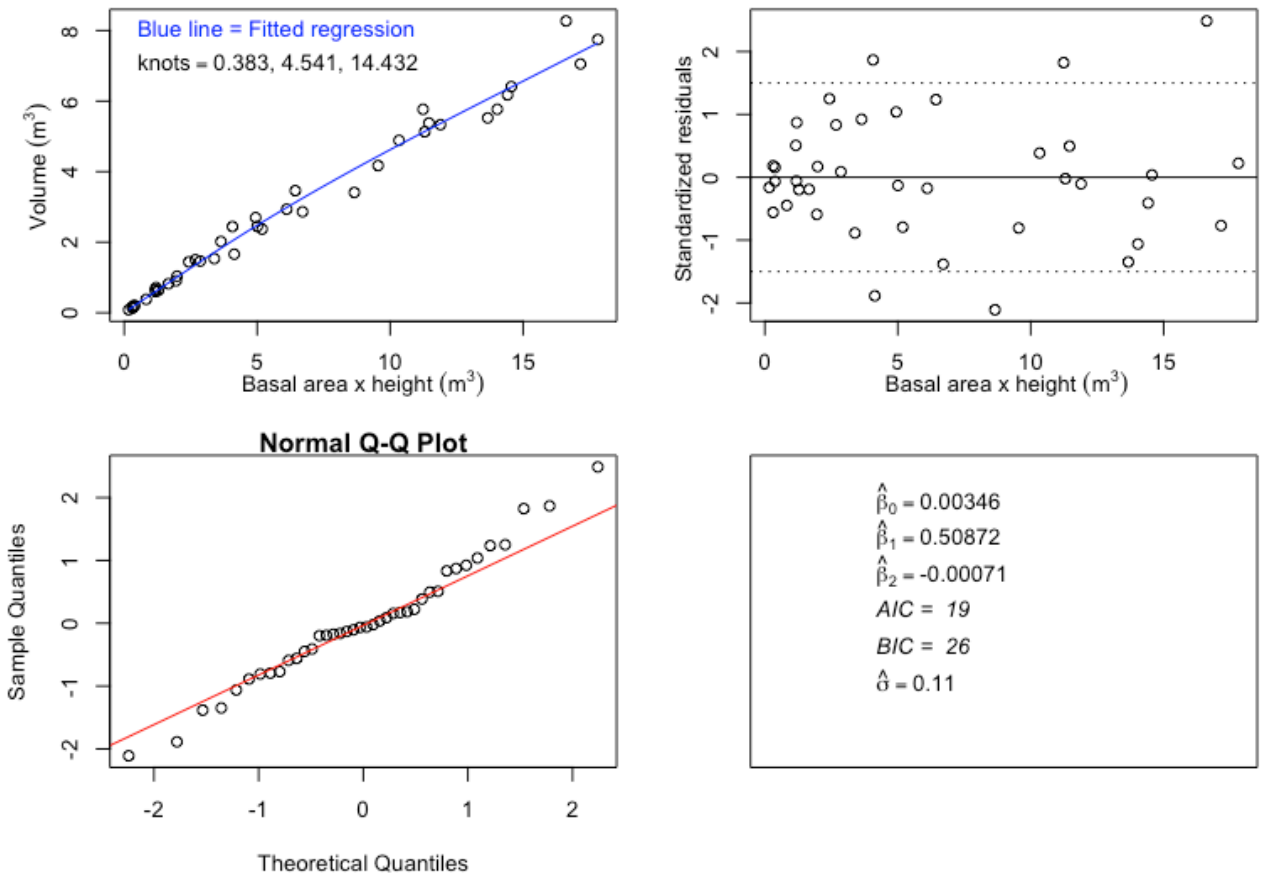
Variance function:
 Structure: fixed weights
 Formula: ~BAH.m3

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	0.0034553	0.026436113	0.130705	0.8967
BAH.m3	0.5087159	0.018467876	27.545990	0.0000
BAH.m3.splinepoints	-0.0007139	0.000212606	-3.357631	0.0018

Plot of Model 14

P_roxburghii:Model 14: (Volume ~ BAH), Cubic spline with varFixed



7.15 Model 15– Volume with basal area * height (BAH) as predictor, with varPower

```
> pr.m15 <- gls(Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints,
  na.action=na.omit, weights = varPower(form =
  ~BAH.m3))
> summary(pr.m15)
```

Generalized least squares fit by REML
 Model: Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints
 Data: NULL
 AIC BIC logLik
 11.27653 19.33112 -0.6382652

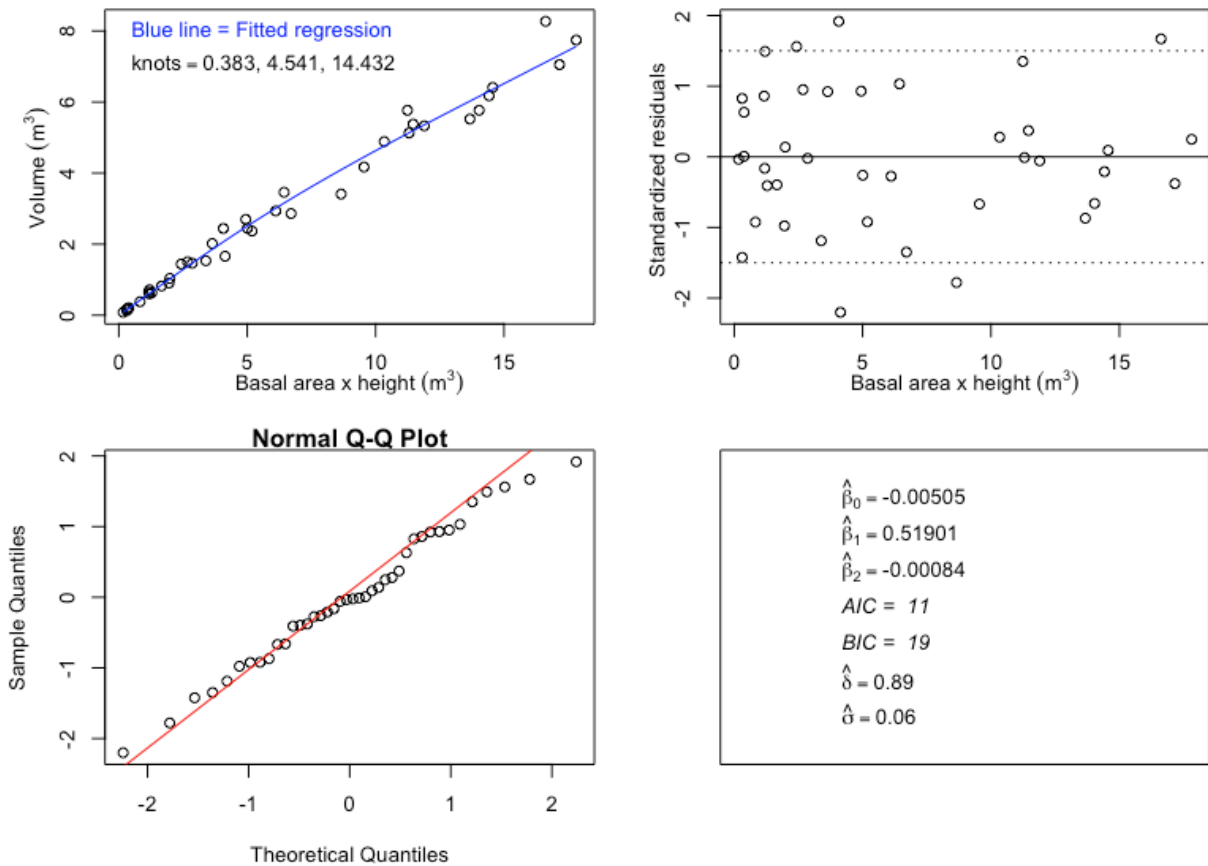
Variance function:
 Structure: Power of variance covariate
 Formula: ~BAH.m3
 Parameter estimates:
 power
 0.8935963

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	-0.0050467	0.009373344	-0.53841	0.5935
BAH.m3	0.5190074	0.014293996	36.30947	0.0000
BAH.m3.splinepoints	-0.0008421	0.000210759	-3.99543	0.0003

Plot of Model 15

P_roxburghii:Model 15: (Volume ~ BAH), Cubic spline with varPower



7.16 Model 16 – Volume with basal area * height (BAH) as predictor, with varConstPower

```
> pr.m16 <- gls(Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints,
  na.action=na.omit, weights = varConstPower(form =
    ~BAH.m3))
> summary(pr.m16)
```

Generalized least squares fit by REML
 Model: Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints
 Data: NULL
 AIC BIC logLik
 13.27653 22.94204 -0.6382652

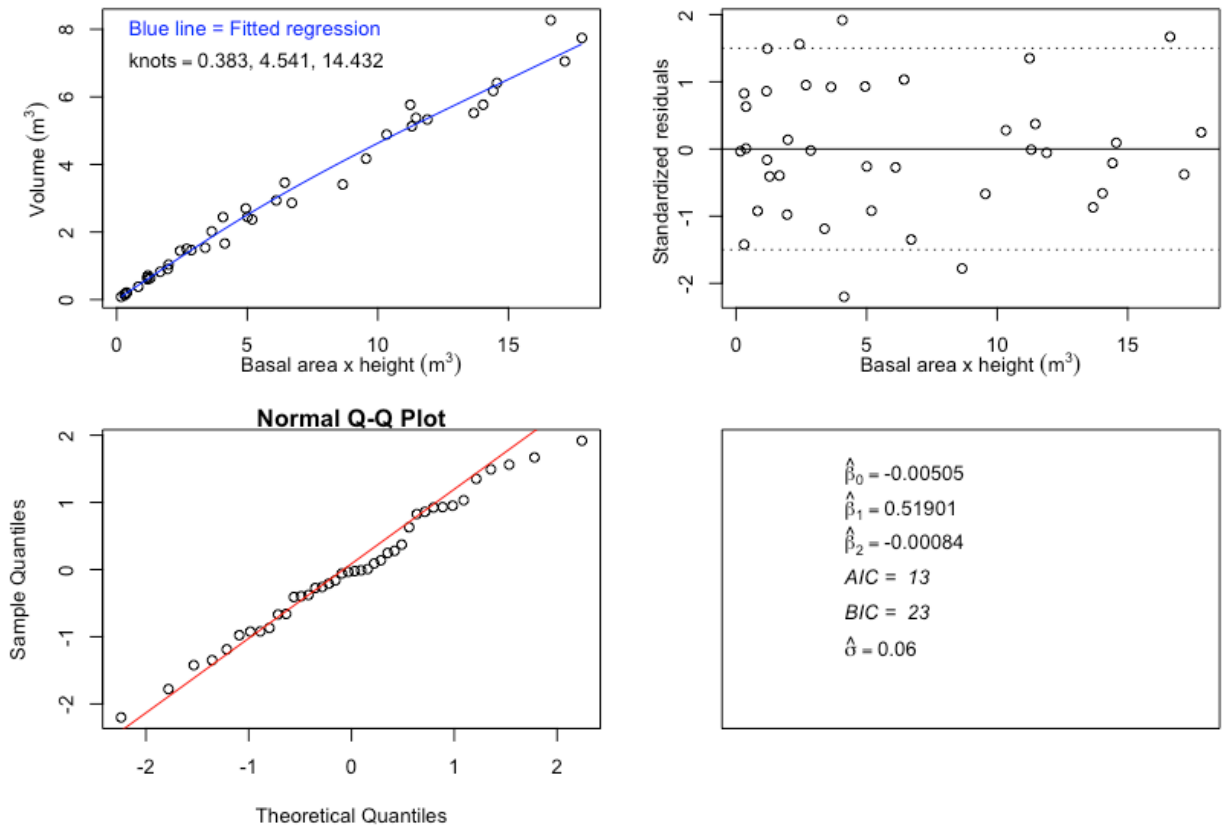
Variance function:
 Structure: Constant plus power of variance covariate
 Formula: ~BAH.m3
 Parameter estimates:
 const power
 5.136601e-10 8.935965e-01

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	-0.0050467	0.009373340	-0.53841	0.5935
BAH.m3	0.5190074	0.014293995	36.30947	0.0000
BAH.m3.splinepoints	-0.0008421	0.000210759	-3.99543	0.0003

Plot of Model 16

P_roxburghii:Model 16: (Volume ~ BAH), Cubic spline with varConstPower



8. Model evaluation using AIC and BIC values

SN	Model	AIC	BIC
1	Model 1 > pr.m1 <- gls(Volume.m3 ~ DBH.cm)	95	100
2	Model 2 > pr.m2 <- gls(Volume.m3 ~ DBH.cm + DBH.cm.splinepoints, na.action=na.omit, weights = varFixed(~DBH.cm))	75	81
3	Model 3 > pr.m3 <- gls(Volume.m3 ~ DBH.cm + DBH.cm.splinepoints, na.action=na.omit, weights = varPower(form = ~DBH.cm))	56	64
4	Model 4 > pr.m4 <- gls(Volume.m3 ~ DBH.cm + DBH.cm.splinepoints, na.action=na.omit, weights = varConstPower(form = ~DBH.cm))	58	68
5	Model 5 > pr.m5 <- gls(Volume.m3 ~ BA.m2)	58	62
6	Model 6 > pr.m6<- gls(Volume.m3 ~ BA.m2 + BA.m2.splinepoints, na.action=na.omit, weights = varFixed(~BA.m2))	22	28
7	Model 7 > pr.m7 <- gls(Volume.m3 ~ BA.m2 + BA.m2.splinepoints, na.action=na.omit, weights = varPower(form = ~BA.m2))	9	17
8	Model 8 > pr.m8 <- gls(Volume.m3 ~ BA.m2 + BA.m2.splinepoints, na.action=na.omit, weights = varConstPower(form = ~BA.m2))	11	21
9	Model 9 > pr.m9 <- gls(Volume.m3 ~ DBH2H.m3)	38	43
10	Model 10 > pr.m10 <-gls(Volume.m3 ~ DBH2H.m3 + DBH2H.m3.splinepoints, na.action=na.omit, weights = varFixed(~DBH2H.m3))	21	28

11	Model 11 > pr.m11 <-gls(Volume.m3 ~ DBH2H.m3 + DBH2H.m3.splinepoints, na.action=na.omit, weights = varPower(form = ~DBH2H.m3))	13	21
12	Model 12 > pr.m12 <- gls(Volume.m3 ~ DBH2H.m3 + DBH2H.m3.splinepoints, na.action=na.omit, weights = varConstPower(form = ~DBH2H.m3))	15	25
13	Model 13 > pr.m13 <- gls(Volume.m3 ~ BAH.m3)	38	42
14	Model 14 > pr.m14 <- gls(Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints, na.action=na.omit, weights = varFixed(~BAH.m3))	19	26
15	Model 15 > pr.m15 <- gls(Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints, na.action=na.omit, weights = varPower(form = ~BAH.m3))	11	19
16	Model 16 > pr.m16 <- gls(Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints, na.action=na.omit, weights = varConstPower(form = ~BAH.m3))	13	23

9. Selected Models

The best fitting models have been selected based on Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) values of the fitted models. The BIC value was mainly relied upon as it imposes a stronger penalty for the number of parameters in the model that need to be estimated. Smaller the values of AIC and BIC, better the fit of the model. Therefore, for *Pinus roxburghii*, the selected models are;

1. Model 7 (Model which doesn't use height)


```
pr.m7 <- gls(Volume.m3 ~ BA.m2 + BA.m2.splinepoints,
              na.action=na.omit, weights = varPower(form = ~BA.m2))
```
2. Model 15 (Model which uses the height)


```
pr.m15 <- gls(Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints,
               na.action=na.omit, weights = varPower(form = ~BAH.m3))
```

Two models have been selected for *Pinus roxburghii*, one without height ($X_1 = BA$ which is model 7) and one with the height ($X_1 = BAH$, which is Model 15) as predictor or explanatory variable. Both the models have been fitted with natural (restricted) cubic spline function within a linear model framework. Although, nonlinear models are more flexible, they are more complicated than the linear models. The complications involved and amount of time and efforts spent on fitting nonlinear models often fail to justify by the improvements in the models. Moreover, the models fitted with natural (restricted) cubic spline functions perform well and track the curvilinearity better than nonlinear functions that were examined.

10. Demonstration of use of the selected best fit model

In general, the natural spline predictor with knots represented by t_1 , t_2 and t_3 takes the following form;

$$Y = \beta_0 + \beta_1 X + \beta_2 X_s + \varepsilon \quad (8)$$

Where X_s corresponds to value in X as follows:

$$X_s = g(X) = (X - t_1)_+^3 - (X - t_2)_+^3 \frac{(t_3 - t_1)}{(t_3 - t_2)} + (X - t_3)_+^3 \frac{(t_2 - t_1)}{(t_3 - t_2)} \quad (9)$$

and the value of the positive part functions depend on the values of the knots as follows;

$$(X - t_1)_+^3 = (X - t_1)_+^3, \text{ if } X > t_1 \text{ and } (X - t_1)_+^3 = 0, \text{ if } X < t_1 \quad (10)$$

$$(X - t_2)_+^3 = (X - t_2)_+^3, \text{ if } X > t_2, \text{ and } (X - t_2)_+^3 = 0, \text{ if } X < t_2 \quad (11)$$

$$(X - t_3)_+^3 = (X - t_3)_+^3, \text{ if } X > t_3, \text{ and } (X - t_3)_+^3 = 0, \text{ if } X < t_3 \quad (12)$$

Where t_1 , t_2 and t_3 for the above models are 10th, 50th and 90th percentiles and are called knots. The values of knots differ from species and models.

To demonstrate use of the selected models for *Pinus roxburghii* – model 7, the knots t1, t2 and t3 are 0.029, 0.166 and 0.421 as generated by the model. The model 7 has been fitted with volume as function of basal area in meter square (BA) i.e

$$BA = \pi r^2 \quad (13)$$

where in

$$r^2 = \left[\frac{dbh}{2 \times 100} \right]^2 \quad (14)$$

Where r is radius in meters and dbh is diameter at breast height in centimeters.

Therefore, *Pinus roxburghii* with diameter of 37 cm resulting in basal area of 0.107521009 m², the volume can be estimated using the above equation (model 7) as below. But first the value of BA.m2 has to be calculated, which is;

$$\begin{aligned} BA &= \pi r^2 = \frac{\pi * 37^2}{200^2} = 0.107521009 \text{ m}^2 \\ g(X) &= (X - t1)_+^3 - (X - t2)_+^3 \frac{(t3-t1)}{(t3-t2)} + (X - t3)_+^3 \frac{(t2-t1)}{(t3-t2)} \\ g(BA) &= (BA - t1)_+^3 - (BA - t2)_+^3 \frac{(t3-t1)}{(t3-t2)} + (BA - t3)_+^3 \frac{(t2-t1)}{(t3-t2)} \\ g(BA) &= (0.107521009 - 0.029)_+^3 - 0 + 0 \\ &= (0.078521009)_+^3 - 0 + 0 \\ &= 0.000484125 \end{aligned}$$

Hence, the volume predicted for this tree by the selected model (model 7) is

$$\begin{aligned} V &= \beta_0 + \beta_1 \cdot BA + \beta_2 BA \cdot m_2 + \varepsilon \\ &= -0.151034 + 14.256793 * 0.107521009 + 27.635236 * 0.000484125 \\ &= -0.151034 + 1.532905 + 0.0133789 \\ &= \mathbf{1.39525 \text{ m}^3} \end{aligned}$$

Similarly, to demonstrate model 15 with t1, t2 and t3 of 0.383, 4.541, and 14.432 respectively, we considered this same tree but with height, i.e dbh = 37 cm resulting in BA = 0.107521009 m² and height (H) = 22.6 m.

$$\begin{aligned} BAH &= 0.107521009 \times 22.6 \\ &= 2.4299748 \\ g(X) &= (X - t1)_+^3 - (X - t2)_+^3 \frac{(t3-t1)}{(t3-t2)} + (X - t3)_+^3 \frac{(t2-t1)}{(t3-t2)} \\ g(BAH) &= (BAH - t1)_+^3 - (BAH - t2)_+^3 \frac{(t3-t1)}{(t3-t2)} + (BAH - t3)_+^3 \frac{(t2-t1)}{(t3-t2)} \\ &= (2.4299748 - 0.383)_+^3 - 0 + 0 \\ &= (2.0469748)_+^3 \\ &= 8.5770411 \end{aligned}$$

Hence, the volume predicted by model 15 for this tree is;

$$\begin{aligned}
 V &= \beta_0 + \beta_1 \cdot BAH.m3 + \beta_2 BAH.m3_2 + \varepsilon \\
 &= -0.0050467 + 0.5190074 * 2.4299748 + (-0.0008421 * 8.5770411) \\
 &= -0.0050467 + 1.261175 + (-0.007223) \\
 &= -0.0050467 + 3.05 - 0.24 \\
 &= \mathbf{1.248905 \text{ m}^3}
 \end{aligned}$$

The field measured volume for this particular tree with DBH of 37 cm and height of 22.6 m is 1.248905 m³.

11. Model Performance

To assess the performance of selected models, we compared the volume predicted by selected models (7 and 15) with the volume of the tree as measured in the field. Using the equations of the selected models, volume prediction or estimation was done in R.

SN	Tree_ID	Height (in m)	DBH (in cm)	Volume in m ³ (Field measured) [A]	Predicted Volume Model_7 [B]	Predicted Volume Model_15 [C]	Difference (Field - Model_7) [A - B]	Difference (Field - Model_15) [A - C]
1	prwc01	22.6	37	1.44194091	1.3952497	1.24890548	0.046691215	0.193035433
2	prwc02	24.5	46	2.44209044	2.28966239	2.06591165	0.152428049	0.376178786
3	prwc03	21.6	26.6	0.71573194	0.64175769	0.61748262	0.073974251	0.098249324
4	prwc04	26.7	55.4	3.46151755	3.53112799	3.1566981	-0.069610443	0.304819448
5	prwc05	13.5	19	0.20855421	0.253187	0.19361054	-0.044632789	0.014943665
6	prwc07	25.6	75.5	5.37867069	7.31077405	5.19481935	-1.932103363	0.183851342
7	prwc08	30.5	68.5	5.76694296	5.83174951	5.11053589	-0.064806545	0.656407068
8	pre01	37.47	77.8	7.74458705	7.82887469	7.56335923	-0.084287642	0.18122782
9	pre02	36.9	57.4	4.17151	3.83950579	4.45254103	0.332004212	-0.281031031
10	pre04	39.4	62	5.32993392	4.60825257	5.35956346	0.721681354	-0.029629538
11	pre05	23.94	32.5	1.03642439	1.0360178	1.02223685	0.000406587	0.014187537
12	pre06	37.1	68.5	5.52193416	5.83174951	6.0253215	-0.309815345	-0.50338734
13	pre07	27.9	47.8	2.4487711	2.50136966	2.51033927	-0.052598561	-0.061568169
14	pre08	36.8	59.8	4.88744858	4.23029148	4.76178767	0.657157097	0.125660906
15	pre09	29.4	39.7	2.01845706	1.63728868	1.85470231	0.381168381	0.163754753
16	pre10	40.41	67.4	6.17483323	5.61387297	6.30233394	0.560960257	-0.127500706
17	pre11	37.63	76.2	7.05233304	7.46681108	7.32112577	-0.414478037	-0.268792728
18	pre13	13.16	19.2	0.19286857	0.26174179	0.19270571	-0.068873217	0.000162863
19	pre15	21.18	27.8	0.63296676	0.71521516	0.66156931	-0.082248397	-0.028602555
20	pre16	23.1	52.2	2.69780269	3.07033907	2.48091546	-0.372536379	0.216887231
21	pre17	25.8	40.9	1.53357203	1.75171232	1.73132371	-0.218140285	-0.197751679
22	pre18	19.9	23	0.38040394	0.44135556	0.4239932	-0.060951616	-0.043589255
23	pre20	34.79	73	6.41142965	6.76525424	6.35551419	-0.353824588	0.055915458
24	pre21	31.12	50	2.93531604	2.77661313	3.01270775	0.158702906	-0.077391711
25	pre22	27.8	36.2	1.46389068	1.32746262	1.46713261	0.136428063	-0.003241929
26	pre23	41.9	65.3	5.76794377	5.20993671	6.15914711	0.558007063	-0.391203339
27	pre24	28.65	54.6	2.86230545	3.41217602	3.27559048	-0.549870567	-0.413285033
28	pre28	13.99	16.8	0.17214775	0.16499747	0.15590653	0.007150279	0.016241225
29	pre29	19.8	27.4	0.65572133	0.69035489	0.60048688	-0.034633563	0.05523445
30	pre31	28.83	34.4	1.50378403	1.18122924	1.37542717	0.322554789	0.128356862
31	pre33	24.9	46	1.66008246	2.28966239	2.09808681	-0.629579931	-0.438004345
32	pre34	10.85	14.1	0.08248901	0.07157869	0.08288213	0.010910316	-0.000393122
33	pre35	17.86	29.1	0.6003979	0.79861936	0.61101114	-0.198221461	-0.010613242

34	prec01	25.1	31.5	0.90735332	0.96325156	1.00689083	-0.055898237	-0.099537511
35	prec02	34.7	64.4	5.12929119	5.0417728	5.1345332	0.087518388	-0.005242014
36	prec03	26.1	28.5	0.82236773	0.75962748	0.8573379	0.062740246	-0.034970172
37	prec04	17.65	15	0.12872634	0.10090435	0.15683233	0.027821993	-0.028105991
38	prec05	33.65	44.3	2.36971765	2.10056515	2.59381777	0.269152499	-0.224100122
39	prec07	37.32	75.3	8.27102878	7.26645654	7.12017092	1.004572239	1.150857856
40	prec08	37.8	54	3.40957869	3.32460808	4.09442932	0.084970613	-0.684850634
				116.392867	116.332977	116.409687	0.059889831	-0.016820141

From the above table, the difference [A-B] provides difference between the volume measured in the field (actual volume) and the volume predicted by model 7. The figures with negative (-) indicates that the volume has been over-predicted by the model 7 vis-à-vis actual volume of the particular tree. And the figures without negative (-) sign indicates the under prediction of volume by the model 7.

Similarly, the difference [A-C] is the difference between the actual volume and the volume predicted by the model 15. Same explanation is applicable here – the figures with negative sign indicates overprediction of volume by the model and vice-versa, while those figures without (-) are under prediction of volume by the model 15.

Summation of the figures in the difference column results in 0.059889831 and – 0.016820141 for model 7 and model 15 respectively. These indicate that the model 7 under predicts total volume for 40 trees by only 0.059889831 m³, while the model 15 over predicts the total volume of 40 trees by 0.016820141 m³. Therefore, looking this, one may be inclined to conclude that overall, model 15 predicts better than model 7, despite model 7 having lower AIC and BIC than model 15.

12. Limitations of the model

The model has the following limitations;

1. The modeling has been done based on only 40 sample trees. The model can be further improved by increasing the number of samples.
2. The diameter for the sample trees ranges between minimum of 14 cm to 77 cm (over bark). Thus, the model prediction for trees above 77 cm must be done with caution, since there were no trees above 77 cm diameter at breast height in the sample.

13. Conclusion

The model 15 that uses the height seems to perform slightly better than the model 7 that doesn't use the height, as empirically shown above. This further reinforces and confirms the observations made by Professor Timothy Gordon Gregoire and Mr. Yograj Chettri while modeling conifer species for biomass estimation. They too observed that in conifers, the models fitted with height as predictors predicted the biomass better than those models that didn't use height as predictor variable.

However, since two models are fitted using different predictors (one with and other without height), it leads us to confidently conclude that *Pinus roxburghii* has two best models, which are;

1. Model 7 - the best fit model which does not use height as a predictor
2. Model 15- the best fit model which uses height as a predictor

14. Acknowledgement

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16. Annexure – Dataset for *Pinus roxburghii*

SN	Tree_ID	Height.m	DBH.cm	Volume.m3	BA.m2	BAH.m3	DBH2H.m3
1	prwc01	22.6	37	1.44194091	0.10752101	2.4299748	3.09394
2	prwc02	24.5	46	2.44209044	0.16619025	4.0716612	5.1842
3	prwc03	21.6	26.6	0.71573194	0.05557163	1.2003473	1.5283296
4	prwc04	26.7	55.4	3.46151755	0.24105126	6.4360687	8.1946572
5	prwc05	13.5	19	0.20855421	0.02835287	0.3827638	0.48735
6	prwc07	25.6	75.5	5.37867069	0.44769659	11.4610327	14.59264
7	prwc08	30.5	68.5	5.76694296	0.36852845	11.2401178	14.3113625
8	pre01	37.47	77.8	7.74458705	0.47538894	17.8128237	22.6799915
9	pre02	36.9	57.4	4.17151	0.25876985	9.5486073	12.1576644
10	pre04	39.4	62	5.32993392	0.30190705	11.8951379	15.14536
11	pre05	23.94	32.5	1.03642439	0.08295768	1.9860069	2.5286625
12	pre06	37.1	68.5	5.52193416	0.36852845	13.6724056	17.4082475
13	pre07	27.9	47.8	2.4487711	0.17945091	5.0066805	6.3747036
14	pre08	36.8	59.8	4.88744858	0.28086152	10.3357041	13.1598272
15	pre09	29.4	39.7	2.01845706	0.12378582	3.6393031	4.6337046
16	pre10	40.41	67.4	6.17483323	0.35678754	14.4177843	18.3572932
17	pre11	37.63	76.2	7.05233304	0.45603673	17.1606622	21.8496337
18	pre13	13.16	19.2	0.19286857	0.02895292	0.3810204	0.4851302
19	pre15	21.18	27.8	0.63296676	0.06069871	1.2855987	1.6368751
20	pre16	23.1	52.2	2.69780269	0.21400843	4.9435948	6.2943804
21	pre17	25.8	40.9	1.53357203	0.13138219	3.3896605	4.3158498
22	pre18	19.9	23	0.38040394	0.04154756	0.8267965	1.05271
23	pre20	34.79	73	6.41142965	0.41853868	14.5609607	18.539591
24	pre21	31.12	50	2.93531604	0.19634954	6.1103977	7.78
25	pre22	27.8	36.2	1.46389068	0.10292172	2.8612237	3.6430232
26	pre23	41.9	65.3	5.76794377	0.33490085	14.0323454	17.8665371
27	pre24	28.65	54.6	2.86230545	0.23413976	6.7081041	8.5410234
28	pre28	13.99	16.8	0.17214775	0.02216708	0.3101174	0.3948538
29	pre29	19.8	27.4	0.65572133	0.05896455	1.1674981	1.4865048
30	pre31	28.83	34.4	1.50378403	0.09294088	2.6794855	3.4116269
31	pre33	24.9	46	1.66008246	0.16619025	4.1381373	5.26884
32	pre34	10.85	14.1	0.08248901	0.0156145	0.1694173	0.2157088
33	pre35	17.86	29.1	0.6003979	0.0665083	1.1878383	1.5124027
34	prec01	25.1	31.5	0.90735332	0.07793113	1.9560714	2.4905475
35	prec02	34.7	64.4	5.12929119	0.32573289	11.3029314	14.3913392
36	prec03	26.1	28.5	0.82236773	0.06379397	1.6650225	2.1199725
37	prec04	17.65	15	0.12872634	0.01767146	0.3119012	0.397125
38	prec05	33.65	44.3	2.36971765	0.1541336	5.1865958	6.6037788
39	prec07	37.32	75.3	8.27102878	0.44532783	16.6196345	21.1607759
40	prec08	37.8	54	3.40957869	0.2290221	8.6570355	11.02248

