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ཚལ་ལྷན་ཁག། ནགས་ཚལ་དང་གླིང་ག་ཞབས་
ཏོག་ལས་ཁུངས།

Royal Government of Bhutan
Ministry of Agriculture and Forests
Department of Forests and Park Services

SPECIES SPECIFIC VOLUME EQUATION TO ESTIMATE MERCHANTABLE VOLUME

Picea spinulosa

Forest Resources Management Division
Department of Forest and Park Services
Ministry of Agriculture and Forests

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Species specific volume equation to estimate merchantable volume

Picea spinulosa

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1. Summary

The volume equation developed in this study will predict the merchantable volume of *Picea spinulosa*. The merchantability standard adopted for this study are the trees above 10 cm in diameter at breast height (dbh) and top diameter measured up to 10 cm over bark have been considered for volume calculation.

A total of 16 models were fitted. First 4 models were fitted with volume as a function of diameter at breast height (DBH), while models 5 – 8 were fitted with basal area (BA) as the predictor variable. With product of squared diameter at breast height and height (DBH2H) as predictor variable, 4 models, namely the models 9 – 12 were fitted. The last four models, 13 -16 were fitted with product of basal area and height (BAH) as the predictor.

The initial plots of response (volume) variables and predictor (DBH, BA, DBH2H and BAH) variables clearly indicated presence of heteroscedasticity, which has been modeled using variance functions (varFixed, varPower and varConstPower) in gls ( ) function of nlme package.

Of the sixteen, two models viz model 7 (fitted with basal area as predictor) and model 16 (fitted with basal x height as predictor) have been selected as the best fit models for models fitted with and without height as predictors respectively. The model 7 had AIC and BIC values of 120 and 131 respectively, whereas the model 16 had AIC and BIC values of 70 and 83 respectively. Lower the AIC and BIC values, better the fit of the model.

The performance of the selected models was assessed by comparing the actual volume with the volumes predicted by two selected models for each tree. The model 16 was found to be the best fit model for models fitted with height as predictor, while model 7 was found to be the best fit model for models fitted without height for *Picea spinulosa*.

2. Introduction

The volume equations, developed during pre-investment survey (PIS) carried out between 1974-81 predict total tree volume, and not the merchantable volume of trees. The recent change of policy of the Department of Forests and Park Services to allot timber for rural house construction in the form of log volume instead of allotting by number of trees as was once practiced, has necessitated development of merchantable log volume equation.

Therefore, standards of merchantability adopted for this study to develop merchantable log volume are all trees above 10 cm diameter at breast height (dbh) and the sections up to 10 cm top diameter over the bark.

As was done for PIS exercise to develop volume equation, this study ignores/does not consider the volume of foliage and branches for the purpose of calculating the merchantable volume. This decision stems from the objective, which is to estimate merchantable log volume. Moreover, branches are rarely used as timber (at least in Bhutan) and are mostly used for firewood.

The sample trees for this study have been felled as part of biomass equation development field work. The data protocol for biomass equation development required collecting a minimum of 8 trees each from four regions of Bhutan namely, eastern, eastern central, western and western central. Therefore, 41 trees in total have been felled for *Picea spinulosa* from three regions namely; east-central, western-central and western regions as part of biomass equation development field work. An additional data for 35 trees has been collected during the months of November, 2018 – January, 2019 from Haa, Paro and Thimphu which fall in the western region. Thus, a total of 76 trees have used in this study.

The trees were felled at 0.3 m height from the ground at which the diameter was measured and recorded. Then diameter at zero height (ground level) were also measured and recorded. After felling, the diameter was measured at 0.7 m from 0.3 m height (essentially making 1 m height, i.e 0.3 m + 0.7 m =1 m). Thereafter, at every meter length, the diameter was measured and recorded, thus making many 1 m length sections of log. As mentioned above the smallest top diameter considered for merchantable log volume calculation was up to 10 cm diameter over bark. Top sections below 10 cm diameter have been discarded.

3. Volume Calculation

Trees after felling are converted into different sizes of sections depending on the requirement and demand. Sections with length of 8 or more feet long are called logs and shorter ones are called sticks or bolts (Avery and Burkhart, 1994). The scaling or measuring the volume of the section is done by multiplying the length with the cross-sectional area of the section. Although they rarely form true circles, they are assumed so for the purpose of calculating cross sectional area in meter square, which is

$$\text{Cross sectional area (A)} = A = \pi r^2 = \frac{\pi D^2}{4*10000} \quad (1)$$

Where **r** is radius in meters and **D** is diameter at breast height in centimeters.

From the ground level to 0.3 m height (height at which sample tree has been cut) is section I, while 0.3 m to 0.7 m is section II. The subsequent sections of 1 m length each are numbered III, IV and so on. The last section is the terminal section, whose length is equal to or less than 1 m. As was adopted for PIS, in this study too the branch volumes are ignored assuming that rarely branches yield merchantable timber.

The most commonly used formulae for calculating volume are the Huber, Newton and Smalian's formulae (Sadiq, 2006, and Goulding, 1979). Of the three commonly used volume calculation approaches or formulae, the Smalian's formula has been used to calculate volume (in m^3) for this study, as under;

$$\text{Section volume } (V_s) = \frac{A+a}{2} * L \quad (2)$$

Where A = Cross sectional area in m^2 at large end of the section
a = Cross sectional area in m^2 at small end of the section
L = Length of the section in meter

Smalian's formula is the easiest and least expensive to apply and therefore applied to get volume for each section of the sample trees. However, for the terminal section, the following formula was used to calculate the volume;

$$\text{Terminal section volume } (V_t) = \frac{A}{3} * L \quad (3)$$

The volume for sections and terminal section for individual trees were then summed to obtain the total volume for each individual sample tree, which is;

$$\text{Volume of tree (V)} = \sum_{s=1}^{n} V_s + V_t \quad (4)$$

After obtaining individual tree volume (Volume.m3), it was then tabulated against the variables - height in meter (Height.m) and the diameter at breast height in centimeter (DBH.cm).

4. The Dataset used for modeling volume of *Picea spinulosa*

A total of 76 trees have been felled and collected data for this study. As part of biomass equation development exercise, 41 trees in total have been felled for *Picea spinulosa* from three regions namely; east-central, western-central and western regions. An additional data for 35 trees has been collected during the months of November, 2018 – January, 2019 from Haa, Paro and Thimphu which fall in the western region. The summary of dataset is presented below, while the detailed dataset is presented as an annexure to this document.

4.1 Summary descriptive statistics of *Picea spinulosa* dataset

```
> summary(ps)
   Tree_ID        Height.m          DBH.cm          Volume.m3
 psec01 : 1    Min.   :10.20    Min.   :10.00    Min.   :0.01825
 psec02 : 1    1st Qu.:22.70    1st Qu.:36.73    1st Qu.:1.17565
 psec05 : 1    Median :29.38    Median :54.65    Median :3.06827
 psec06 : 1    Mean   :29.87    Mean   :51.89    Mean   :3.64047
 psec07 : 1    3rd Qu.:36.73    3rd Qu.:69.08    3rd Qu.:5.45357
 psec08 : 1    Max.   :56.00    Max.   :99.50    Max.   :13.6588

     BA.m2              BAH.m3             DBH2H.m3
 Min.   :0.007854   Min.   : 0.08404   Min.   : 0.107
 1st Qu.:0.105936   1st Qu.: 2.39167   1st Qu.: 3.045
 Median :0.234570   Median : 7.39421   Median : 9.415
 Mean   :0.248304   Mean   : 8.54410   Mean   : 10.879
 3rd Qu.:0.374743   3rd Qu.: 12.97366  3rd Qu.: 16.519
 Max.   :0.777564   Max.   : 27.83372  Max.   : 35.439
```

5. Fitting the models

The models have been fitted in R, which is a robust statistical computing environment. It is a powerful tool which provides wide range of statistical and graphical options to explore, calculate and manage data besides modelling. It is very powerful and widely used statistical tool which is free and allows user to customize the scripts depending on desired output, which is not possible in many of the statistical softwares.

After reading in the excel files into R, we created other variables namely; basal area in square meter (BA.m2), basal area in meter times height in meter (BAH.m3) and square of the diameter in meter times height in meter (DBH2H.m3). The height in meter (Height.m) and diameter in centimeter (DBH.cm) were measured and recorded in the field.

Prior to fitting models, we explored and examined each set of data by preparing descriptive summaries that provided mean, median and range of dependent/response and independent variables. Then we plotted scatter graphs which provided sense of relationship between the dependent/response (volume) and independent/predictor variables (namely DBH.cm, BA.m2, DBH2H.m3 and BAH.m3). These graphs showed curvilinear relationship between response and predictor variables. The scatter plots also clearly revealed the presence of phenomenon, referred in statistical parlance, as heteroscedasticity, which is the increase in variation in response (volume) variable with increase in value of the predictor variables.

Therefore, we fitted the models using the gls ( ) function of the nlme package of R, because the gls ( ) function has the capability to model heteroscedasticity. We didn't transform the variables, mainly response variable, because transformation makes it difficult to directly interpret the relationship between response and predictor variables; and secondly to compare the AIC and BIC values among the different models, the response variables need to be identical.

The models were fitted with volume as a function of four variables;

1) DBH.cm,
2) BA.m2,
3) DBH2H.m3 and
4) BAH.m3.

For each of the variable, we fitted one simple gls ( ) function, which can be written in the following form;

$$Y = \beta_0+\beta_1 X + \varepsilon, \tag{5}$$

Where Y = Volume (V) and X = predictor variable

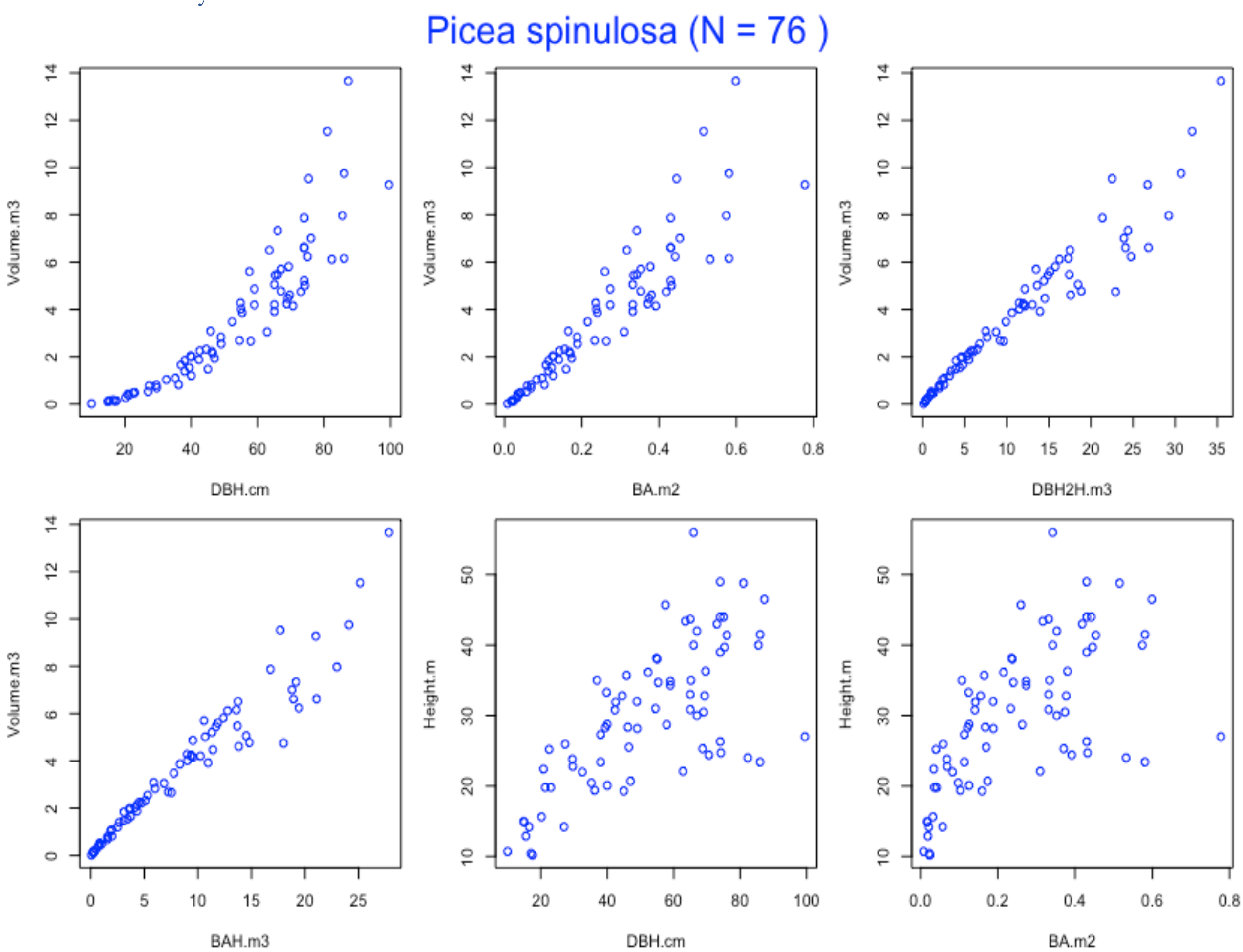
And then fitted 3 models with restricted natural cubic spline functions. The restricted natural cubic spline function enables better tracking of curvilinear relationship between response and predictor variables. These models introduce an additional predictor variable as part of a 3 knot-cubic spline. They take the following forms;

$$Y = \beta_0+\beta_1 X_1 + \beta_2 X_2 + \varepsilon, \tag{6}$$

Where Y = Response variable, volume (V)
X_1 = Predictor variable
$X_2 = g(X_1)$

And $g(X_1)$ is the spline transformation of X_1 predictor variable

1. Summary Plots



6. Models and results

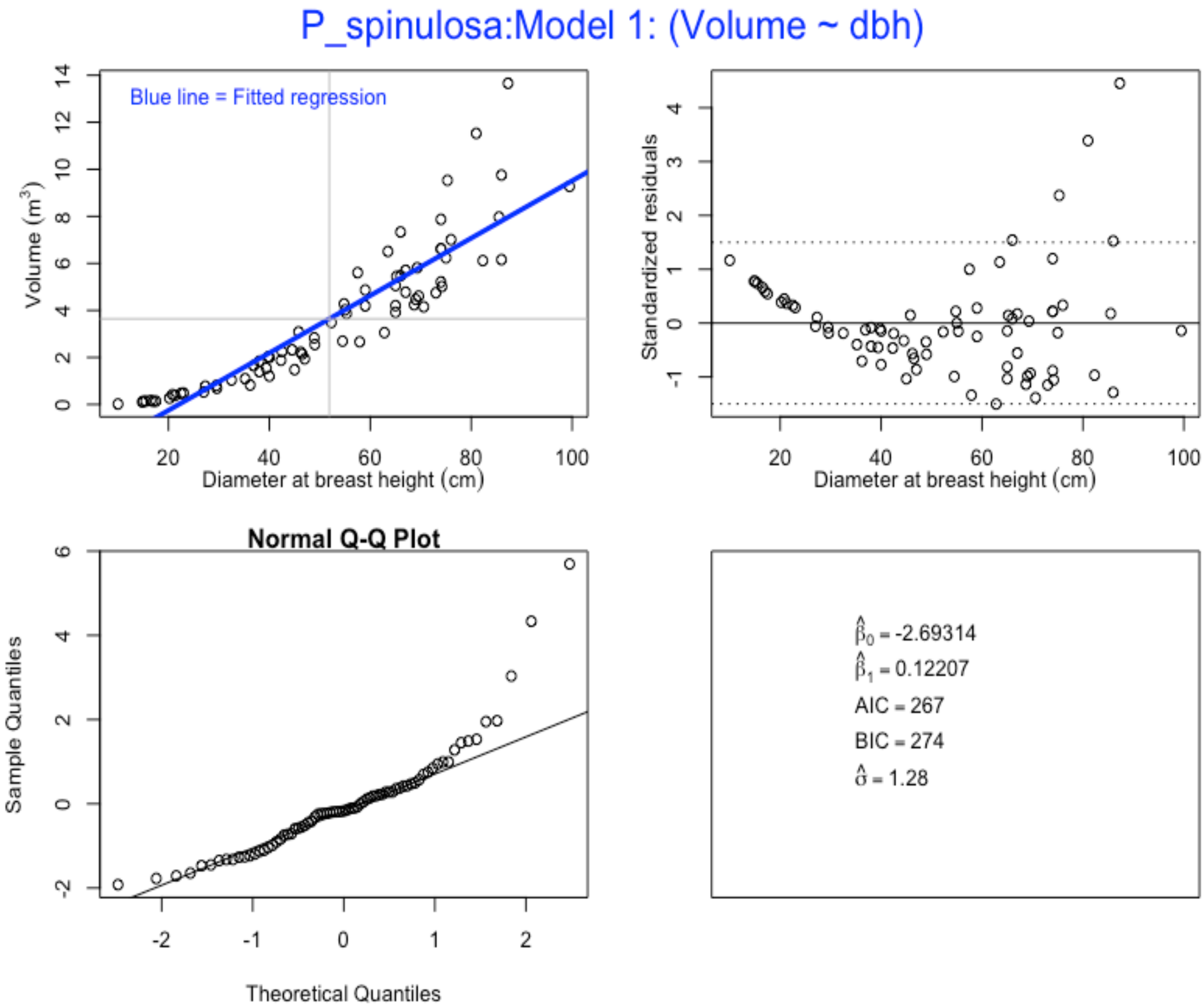
6.1 Model 1 - Volume with diameter at breast height (DBH) as predictor

```
> ps.m1 <- gls(Volume.m3 ~ DBH.cm)
> summary(ps.m1)

Generalized least squares fit by REML
  Model: Volume.m3 ~ DBH.cm
  Data: NULL
       AIC      BIC    logLik
  267.1811 274.0933 -130.5906

Coefficients:
                 Value Std.Error   t-value p-value
(Intercept) -2.6931393 0.3806118 -7.075816       0
DBH.cm       0.1220688 0.0067692 18.033050       0
```

Plot of model 1



6.2 Model 2 - Volume with diameter at breast height (DBH) as predictor, with varFixed

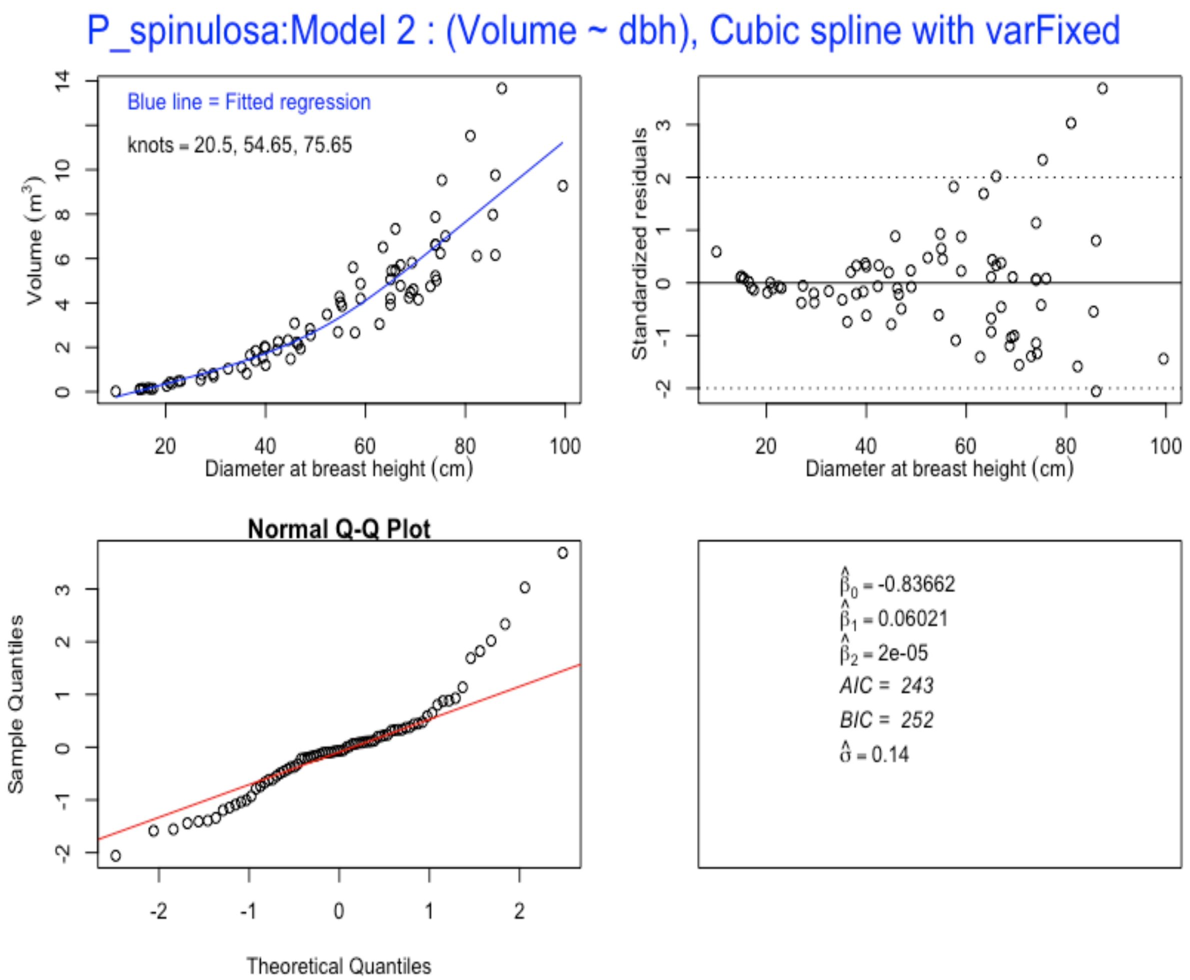
```
> ps.m2 <- gls(Volume.m3 ~ DBH.cm + DBH.cm.splinepoints,
           na.action=na.omit, weights = varFixed(~DBH.cm))
> summary(ps.m2)

Generalized least squares fit by REML
  Model: Volume.m3 ~ DBH.cm + DBH.cm.splinepoints
  Data: NULL
       AIC      BIC    logLik
  243.2867 252.4485 -117.6433

Variance function:
 Structure: fixed weights
 Formula: ~DBH.cm

Coefficients:
                         Value  Std.Error   t-value p-value
(Intercept)         -0.8366246 0.28935098 -2.891383   0.005
DBH.cm               0.0602075 0.00976901  6.163110   0.000
DBH.cm.splinepoints  0.0000218 0.00000392  5.565134   0.000
```

Plot of Model 2



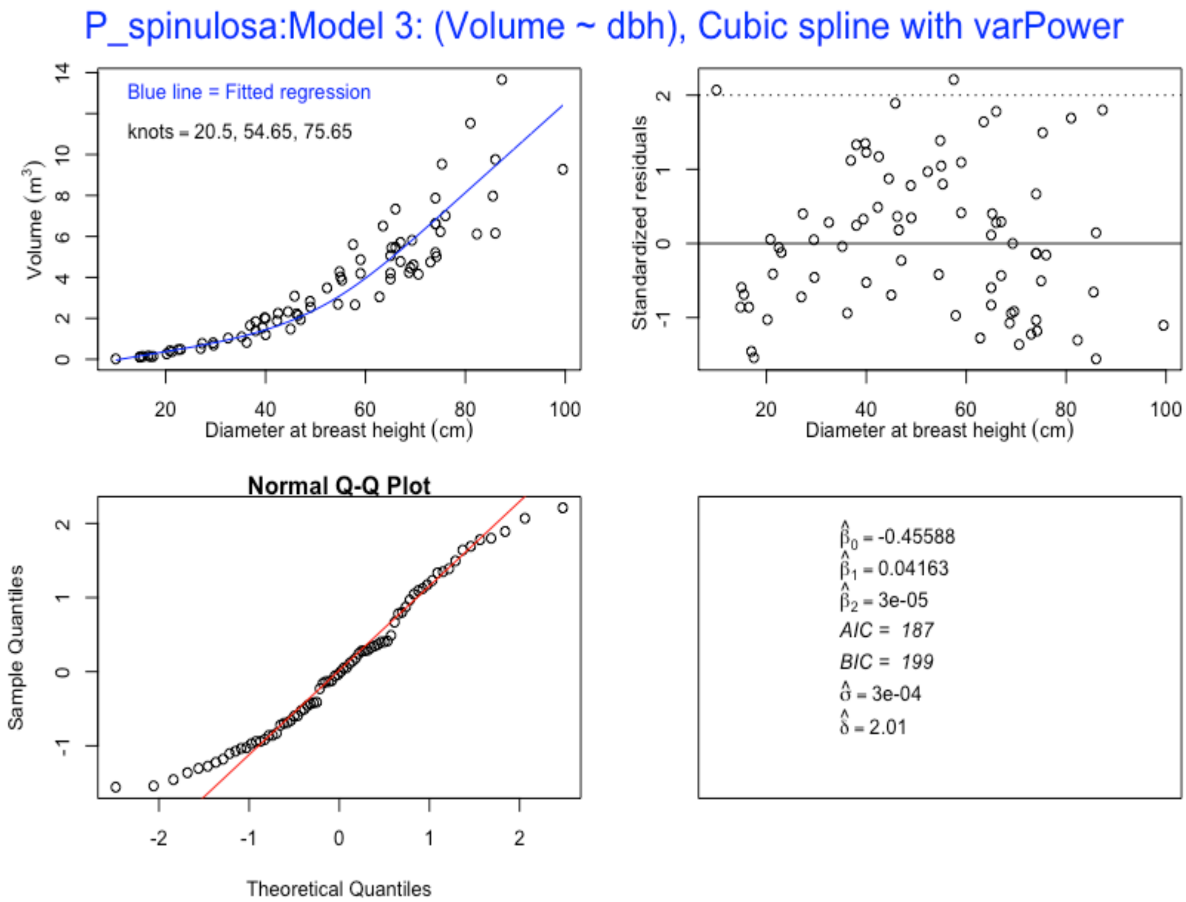
6.3 Model 3- Volume with diameter at breast height (DBH) as predictor, with varPower

```
> ps.m3 <- gls(Volume.m3 ~ DBH.cm + DBH.cm.splinepoints,
             na.action=na.omit, weights = varPower(form =
             ~DBH.cm))
> summary(ps.m3)

Generalized least squares fit by REML
  Model: Volume.m3 ~ DBH.cm + DBH.cm.splinepoints
  Data: NULL
       AIC      BIC    logLik
  187.2164 198.6687 -88.60821

Variance function:
 Structure: Power of variance covariate
 Formula: ~DBH.cm
 Parameter estimates:
  power
2.01027
Coefficients:
                        Value  Std.Error   t-value p-value
(Intercept)        -0.4558764 0.04944989 -9.218957       0
DBH.cm              0.0416262 0.00310185 13.419818       0
DBH.cm.splinepoints 0.0000313 0.00000269 11.632674       0
```

Plot of Model 3



6.4 Model 4 - Volume with diameter at breast height (DBH) as predictor, with varConstPower

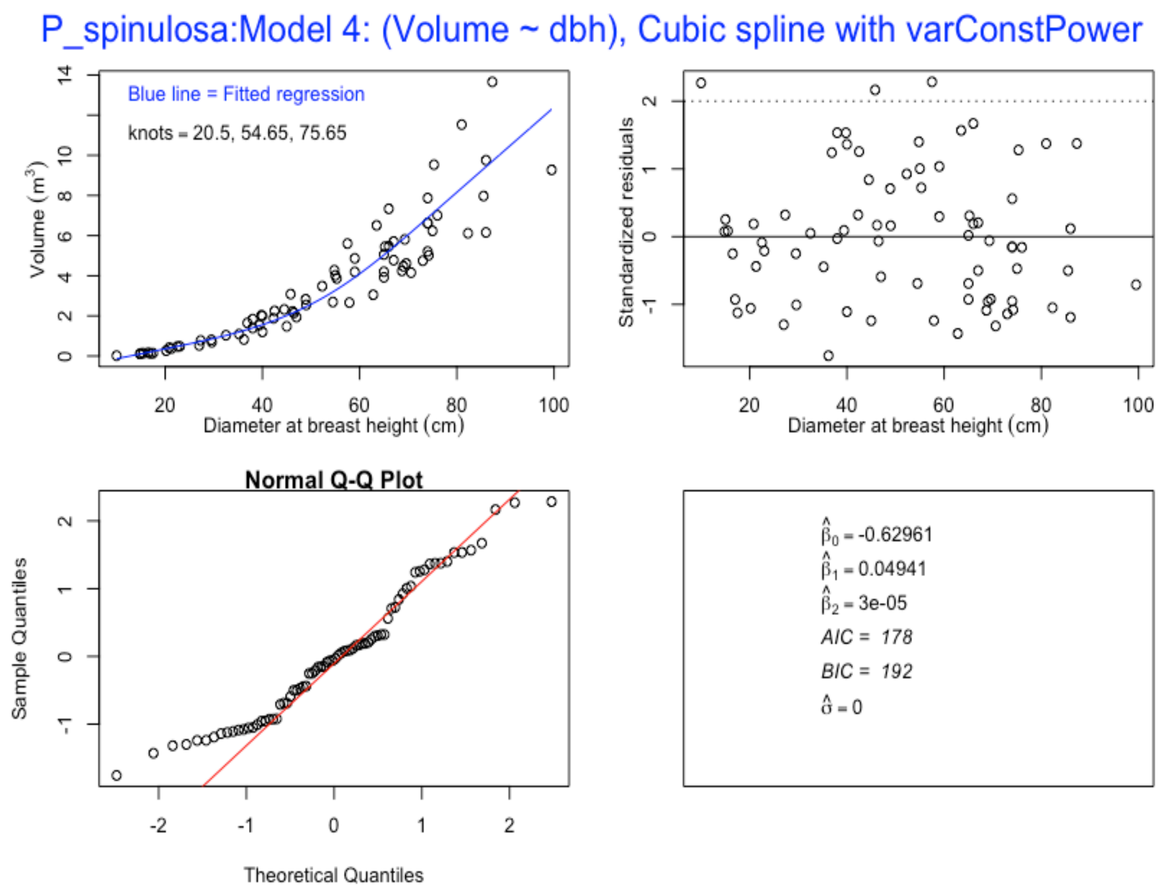
```
> ps.m4 <- gls(Volume.m3 ~ DBH.cm + DBH.cm.splinepoints,
              na.action=na.omit, weights = varConstPower(form =
              ~DBH.cm))
> summary(ps.m4)

Generalized least squares fit by REML
  Model: Volume.m3 ~ DBH.cm + DBH.cm.splinepoints
  Data: NULL
       AIC      BIC    logLik
  177.9158 191.6586 -82.95791

Variance function:
 Structure: Constant plus power of variance covariate
 Formula: ~DBH.cm
 Parameter estimates:
      const       power
19616.922688    3.057581

Coefficients:
                         Value  Std.Error   t-value p-value
(Intercept)         -0.6296128 0.06629718 -9.496826       0
DBH.cm               0.0494055 0.00322285 15.329777       0
DBH.cm.splinepoints  0.0000288 0.00000295  9.769065       0
```

Plot of Model 4



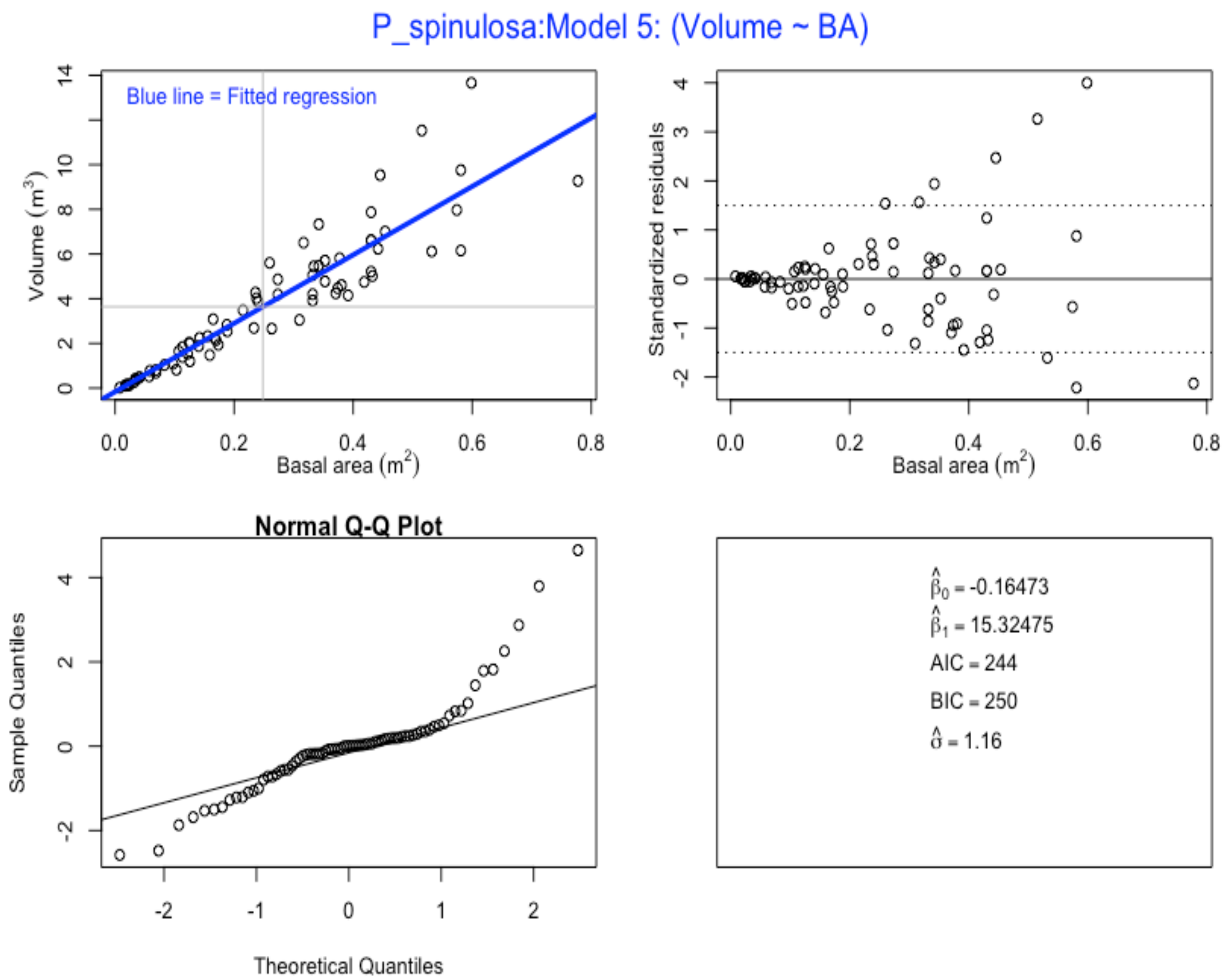
6.5 Model 5 - Volume with basal area (BA) as predictor

```
> ps.m5 <- gls(Volume.m3 ~ BA.m2)
> summary(ps.m5)

Generalized least squares fit by REML
  Model: Volume.m3 ~ BA.m2
  Data: NULL
       AIC      BIC    logLik
  243.5707 250.4829 -118.7854

Coefficients:
                Value Std.Error   t-value p-value
(Intercept) -0.164728 0.2308128 -0.713686  0.4777
BA.m2       15.324750 0.7584809 20.204530  0.0000
```

Plot of Model 5



6.6 Model 6 - Volume with basal area (BA) as predictor, with varFixed

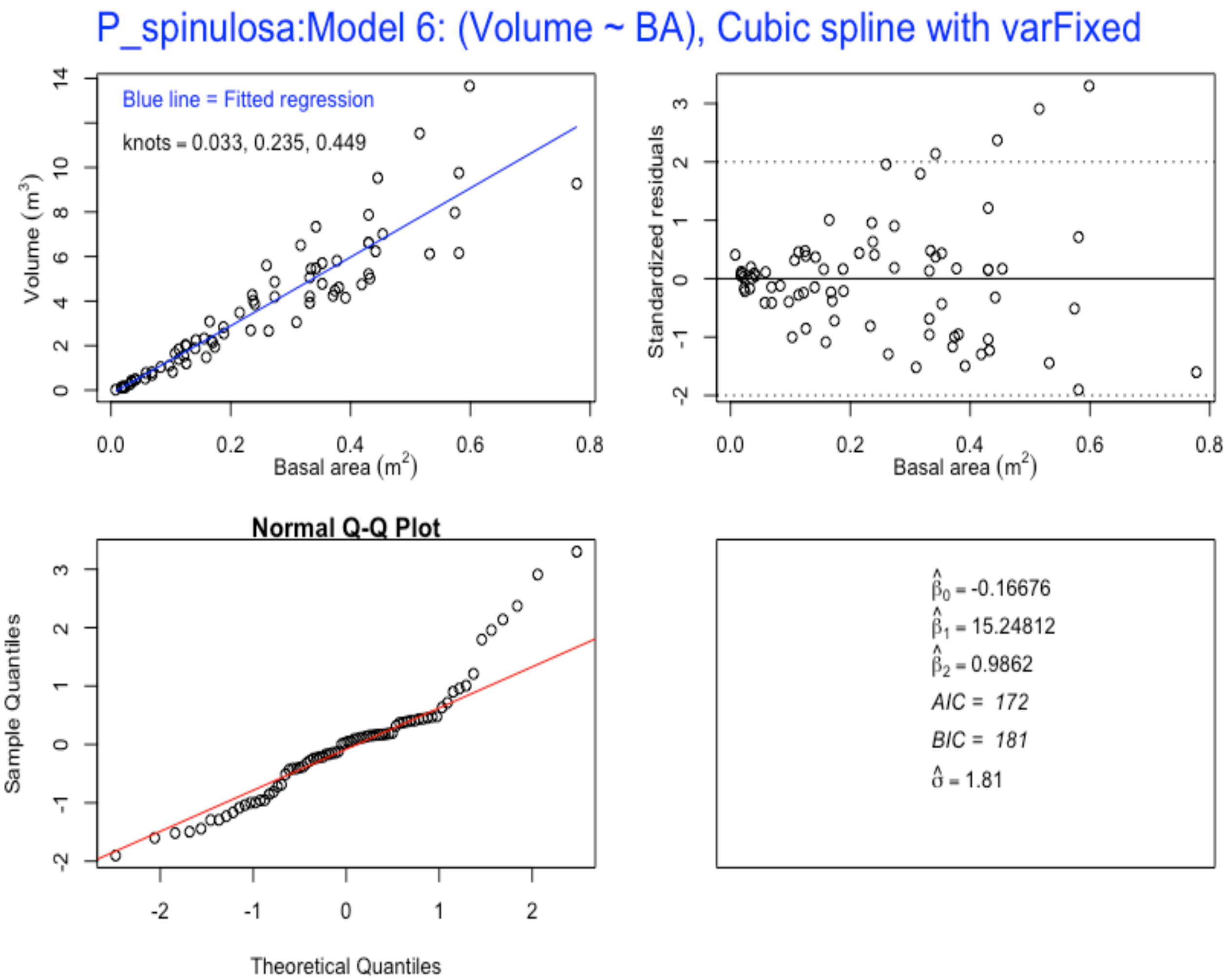
```
> ps.m6<- gls(Volume.m3 ~ BA.m2 + BA.m2.splinepoints, na.action=na.omit, weights =
          varFixed(~BA.m2))
> summary(ps.m6)
```

```
Generalized least squares fit by REML
  Model: Volume.m3 ~ BA.m2 + BA.m2.splinepoints
  Data: NULL
       AIC      BIC    logLik
  171.7423 180.9041 -81.87115

Variance function:
 Structure: fixed weights
 Formula: ~BA.m2

Coefficients:
                       Value Std.Error   t-value p-value
(Intercept)        -0.166761  0.086992 -1.916966  0.0592
BA.m2              15.248122  1.146454 13.300248  0.0000
BA.m2.splinepoints  0.986202  9.818264  0.100446  0.9203
```

Plot of Model 6



6.7 Model 7 Volume with basal area (BA) as predictor, with varPower

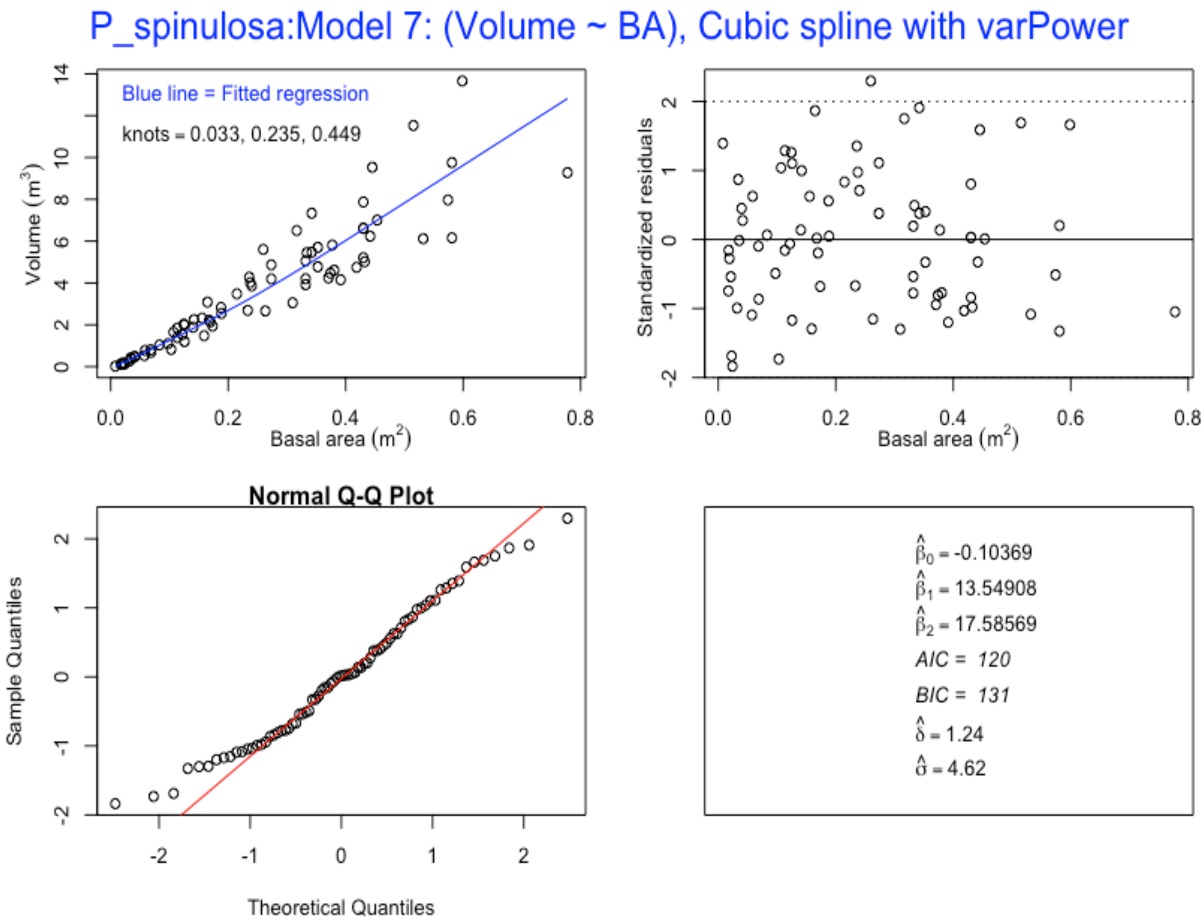
```
> ps.m7 <- gls(Volume.m3 ~ BA.m2 + BA.m2.splinepoints,
           na.action=na.omit, weights = varPower(form = ~BA.m2))
> summary(ps.m7)

Generalized least squares fit by REML
  Model: Volume.m3 ~ BA.m2 + BA.m2.splinepoints
  Data: NULL
       AIC      BIC    logLik
  119.8719 131.3242 -54.93596

Variance function:
 Structure: Power of variance covariate
 Formula: ~BA.m2
 Parameter estimates:
   power
1.243562

Coefficients:
                       Value Std.Error   t-value p-value
(Intercept)        -0.103695  0.012005 -8.637746  0.0000
BA.m2              13.549084  0.546980 24.770703  0.0000
BA.m2.splinepoints 17.585685  8.207566  2.142619  0.0355
```

Plot of Model 7



6.8 Model 8 – Volume with basal area (BA) as predictor, with varConstPower

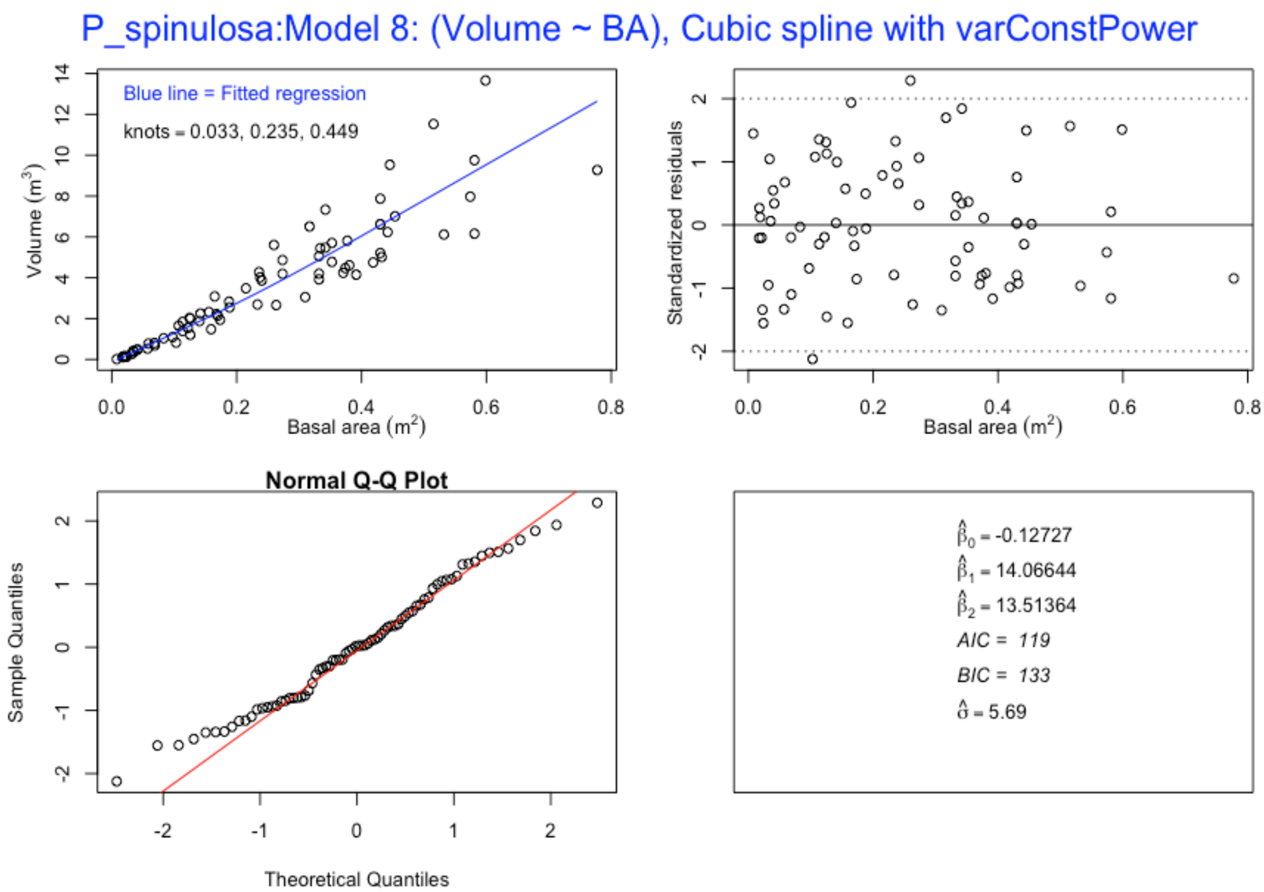
```
> ps.m8 <- gls(Volume.m3 ~ BA.m2 + BA.m2.splinepoints,
             na.action=na.omit, weights = varConstPower(form =
             ~BA.m2))
> summary(ps.m8)
```

```
Generalized least squares fit by REML
  Model: Volume.m3 ~ BA.m2 + BA.m2.splinepoints
  Data: NULL
       AIC      BIC    logLik
  119.3787 133.1214 -53.68933

Variance function:
 Structure: Constant plus power of variance covariate
 Formula: ~BA.m2
 Parameter estimates:
      const       power
0.003305888 1.435174311

Coefficients:
                       Value Std.Error   t-value p-value
(Intercept)        -0.127273  0.017975 -7.080544  0.0000
BA.m2              14.066442  0.566054 24.849996  0.0000
BA.m2.splinepoints 13.513638  8.721142  1.549526  0.1256
```

Plot of Model 8



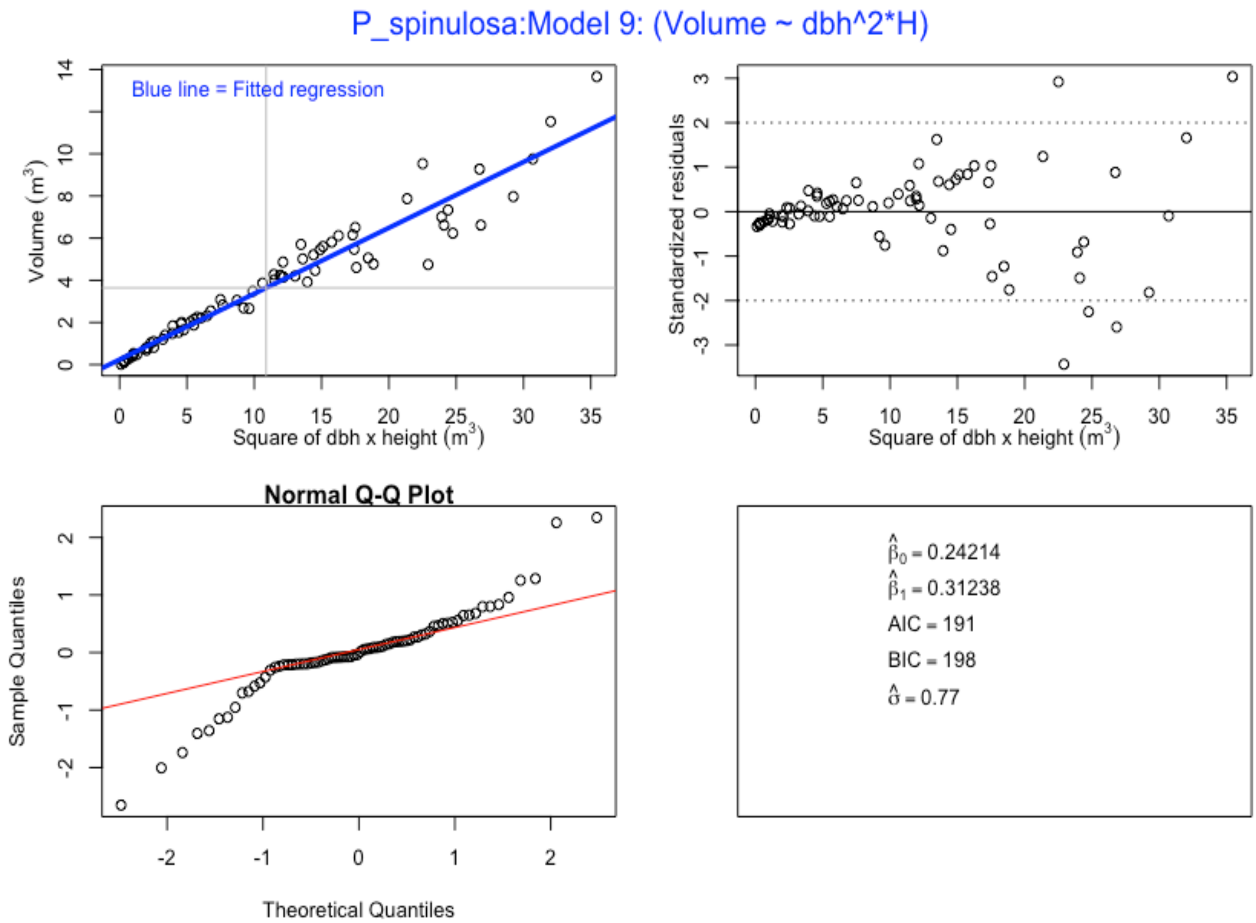
6.9 Model 9 – Volume with square of diameter at breast height * height (DBH2H) as predictor

```
> ps.m9 <- gls(Volume.m3 ~ DBH2H.m3)
> summary(ps.m9)
```

```
Generalized least squares fit by REML
  Model: Volume.m3 ~ DBH2H.m3
  Data: NULL
        AIC      BIC    logLik
  190.9355 197.8477 -92.46773

Coefficients:
                Value  Std.Error  t-value p-value
(Intercept) 0.2421415 0.13855706  1.74759  0.0847
DBH2H.m3    0.3123838 0.00978837 31.91377  0.0000
```

Plot of Model 9



6.10 Model 10 – Volume with square of diameter at breast height * height (DBH2H) as predictor, with varFixed

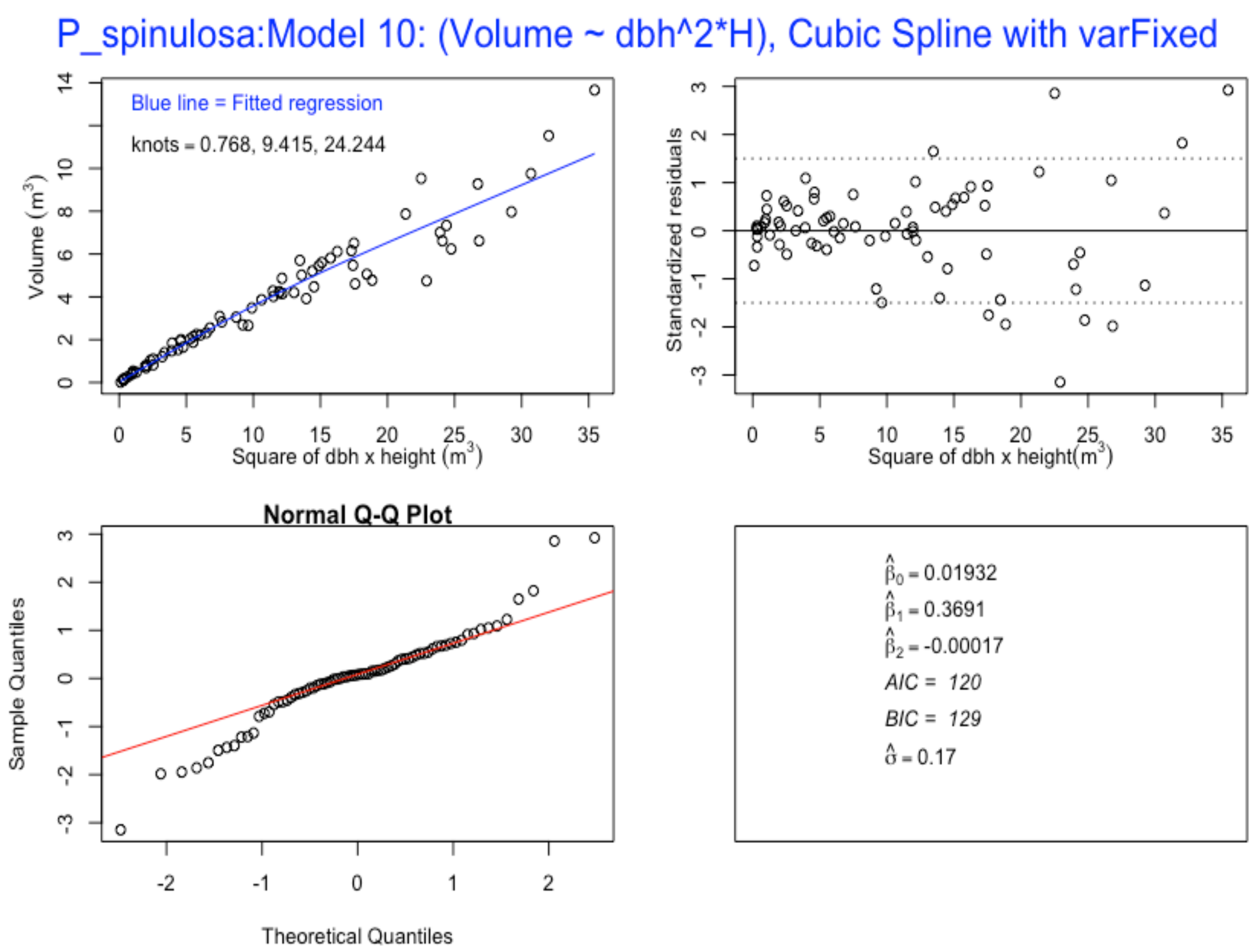
```
> ps.m10 <- gls(Volume.m3 ~ DBH2H.m3 + DBH2H.m3.splinepoints,
                na.action=na.omit, weights = varFixed(~DBH2H.m3))
> summary(ps.m10)

Generalized least squares fit by REML
  Model: Volume.m3 ~ DBH2H.m3 + DBH2H.m3.splinepoints
  Data: NULL
       AIC      BIC    logLik
  119.6496 128.8114 -55.82479

Variance function:
 Structure: fixed weights
 Formula: ~DBH2H.m3

Coefficients:
                            Value   Std.Error   t-value p-value
(Intercept)             0.0193226 0.030648492  0.630458  0.5304
DBH2H.m3                0.3691035 0.013849587 26.650865  0.0000
DBH2H.m3.splinepoints -0.0001656 0.000051196 -3.234859  0.0018
```

Plot of Model 10



6.11 Model 11– Volume with square of diameter at breast height * height (DBH2H) as predictor, with varPower

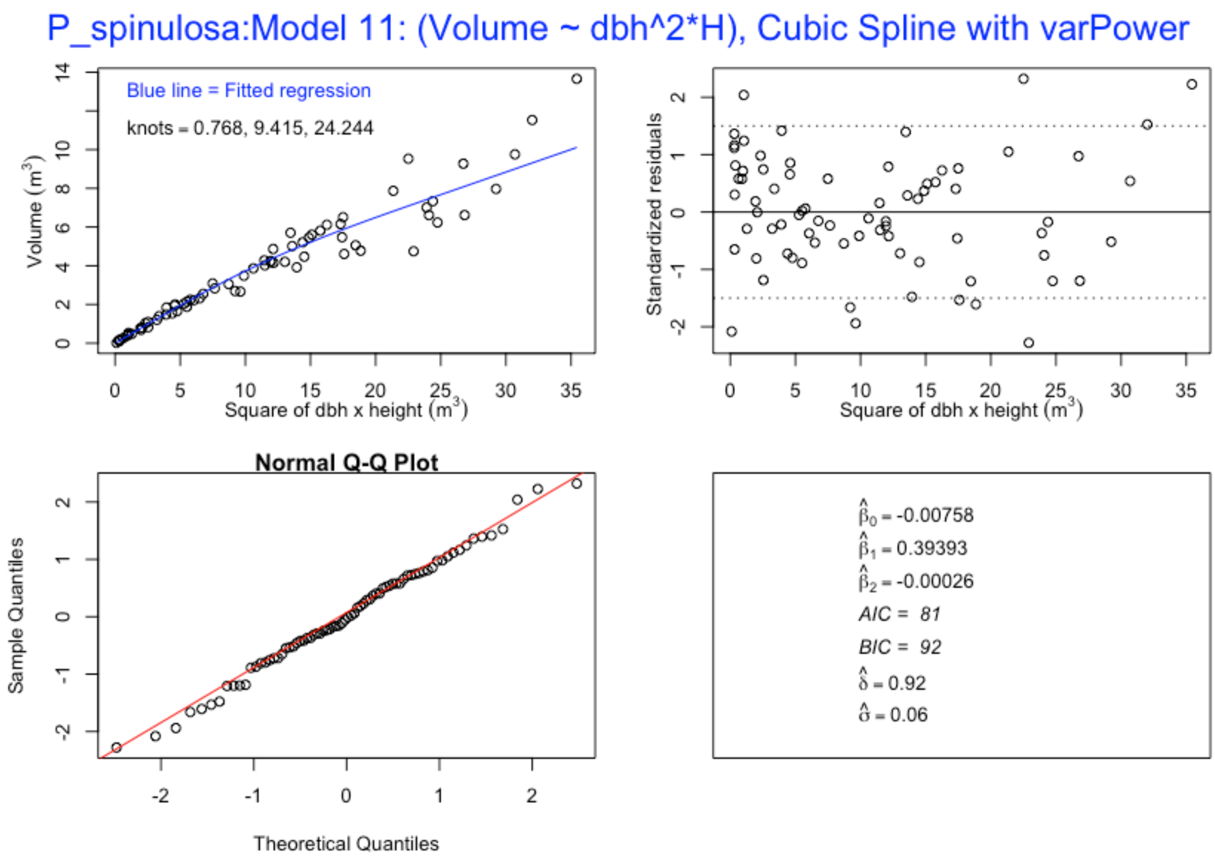
```
> ps.m11 <- gls(Volume.m3 ~ DBH2H.m3 + DBH2H.m3.splinepoints,
na.action=na.omit, weights = varPower(form = ~DBH2H.m3))
> summary(ps.m11)
```

```
Generalized least squares fit by REML
  Model: Volume.m3 ~ DBH2H.m3 + DBH2H.m3.splinepoints
  Data: NULL
       AIC      BIC    logLik
  80.56532 92.01762 -35.28266

Variance function:
 Structure: Power of variance covariate
 Formula: ~DBH2H.m3
 Parameter estimates:
    power
0.9158461

Coefficients:
                           Value   Std.Error  t-value p-value
(Intercept)           -0.0075769 0.006419345 -1.18033  0.2417
DBH2H.m3               0.3939314 0.009853123 39.98036  0.0000
DBH2H.m3.splinepoints -0.0002633 0.000048814 -5.39475  0.0000
```

Plot of Model 11



6.12 Model 12 –Volume with square of diameter at breast height * height (DBH2H) as predictor, with varConstPower

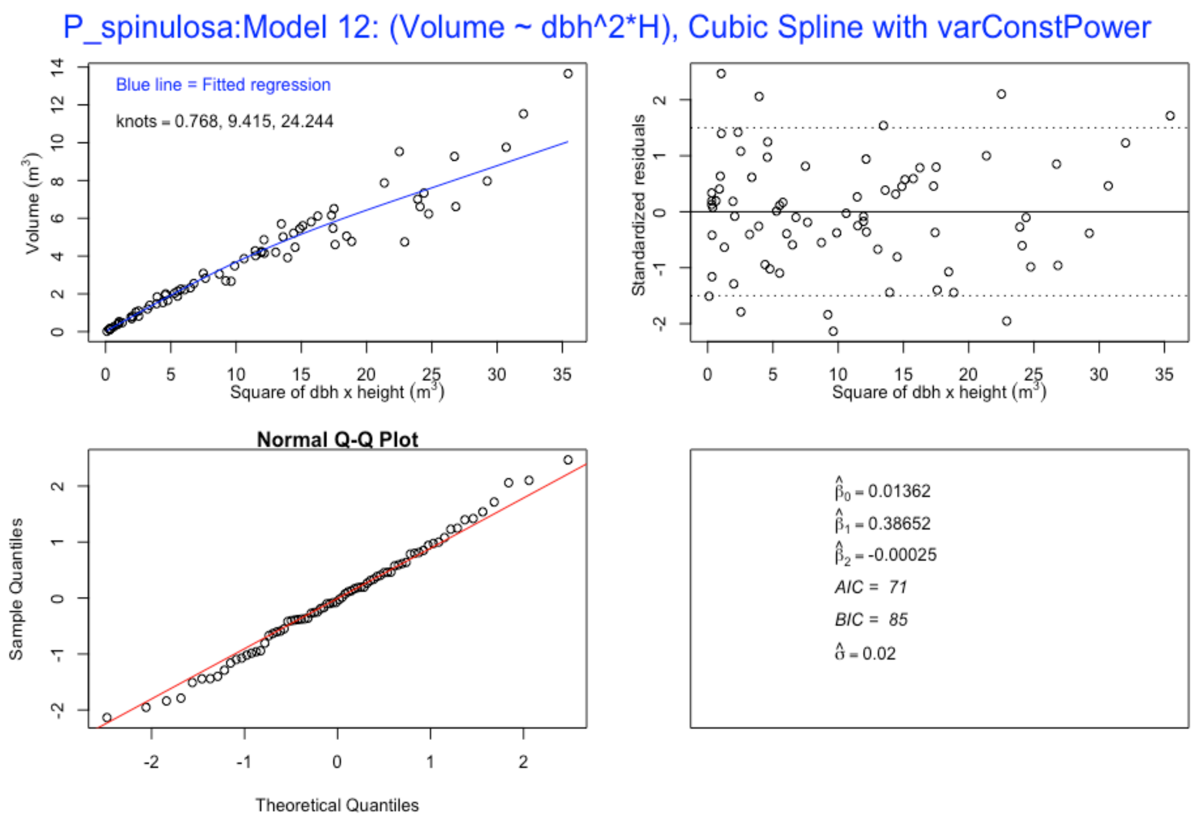
```
> ps.m12 <- gls(Volume.m3 ~ DBH2H.m3 + DBH2H.m3.splinepoints,
              na.action=na.omit, weights = varConstPower(form =
              ~DBH2H.m3))
> summary(ps.m12)
```

```
Generalized least squares fit by REML
  Model: Volume.m3 ~ DBH2H.m3 + DBH2H.m3.splinepoints
  Data: NULL
       AIC      BIC    logLik
  71.46698 85.20973 -29.73349

Variance function:
 Structure: Constant plus power of variance covariate
 Formula: ~DBH2H.m3
 Parameter estimates:
   const    power
1.033237 1.271253

Coefficients:
                           Value   Std.Error  t-value p-value
(Intercept)            0.0136202 0.010590770  1.28604  0.2025
DBH2H.m3               0.3865154 0.008628737 44.79398  0.0000
DBH2H.m3.splinepoints -0.0002506 0.000050333 -4.97869  0.0000
```

Plot of Model 12



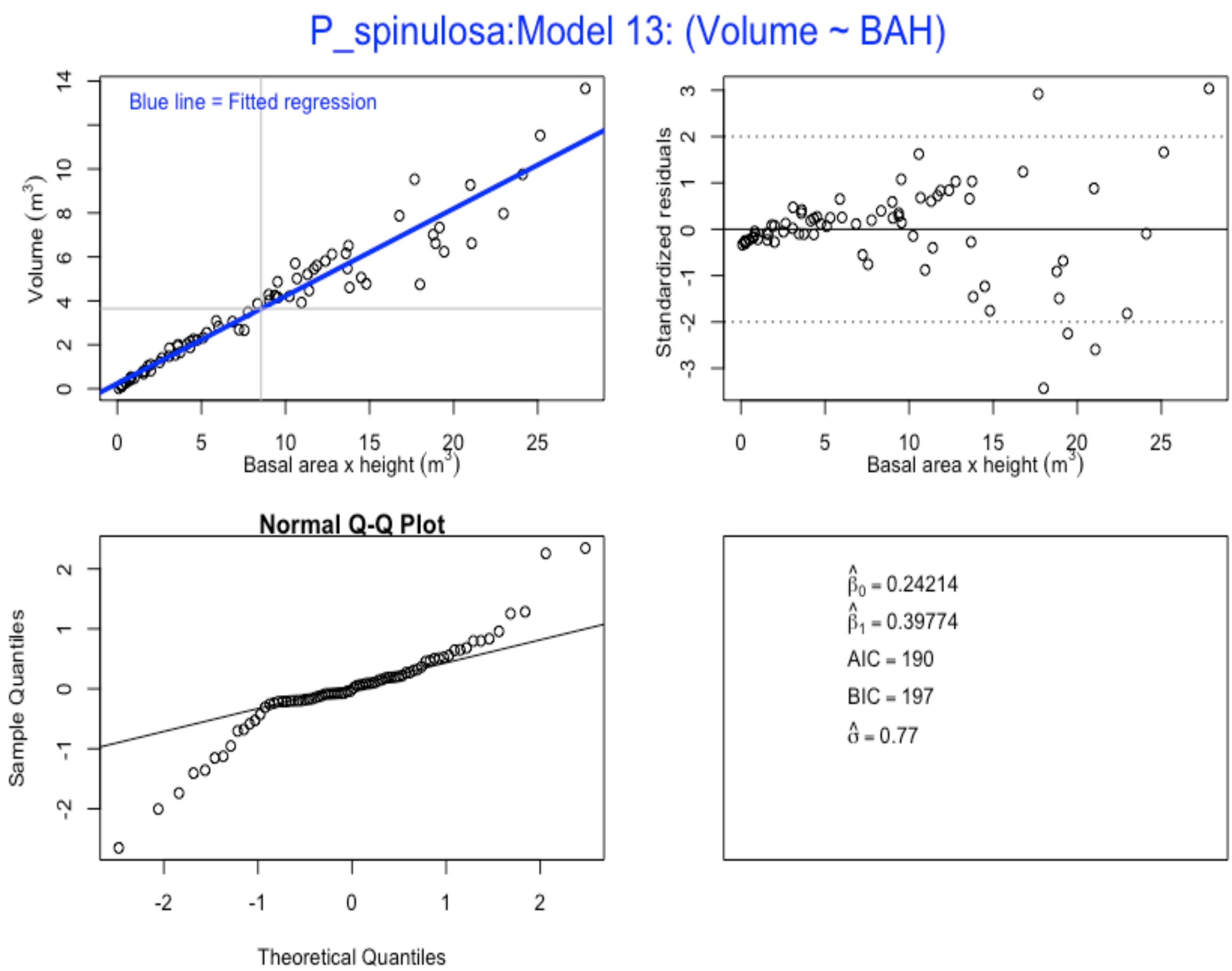
6.13 Model 13 – Volume with basal area * height (BAH) as predictor

```
> ps.m13 <- gls(Volume.m3 ~ BAH.m3)
> summary(ps.m13)
```

```
Generalized least squares fit by REML
  Model: Volume.m3 ~ BAH.m3
  Data: NULL
       AIC      BIC    logLik
  190.4523 197.3645 -92.22616

Coefficients:
                Value  Std.Error  t-value p-value
(Intercept) 0.2421415 0.13855706  1.74759  0.0847
BAH.m3      0.3977393 0.01246294 31.91377  0.0000
```

Plot of Model 13



6.14 Model 14 – Volume with basal area * height (BAH) as predictor, with varFixed

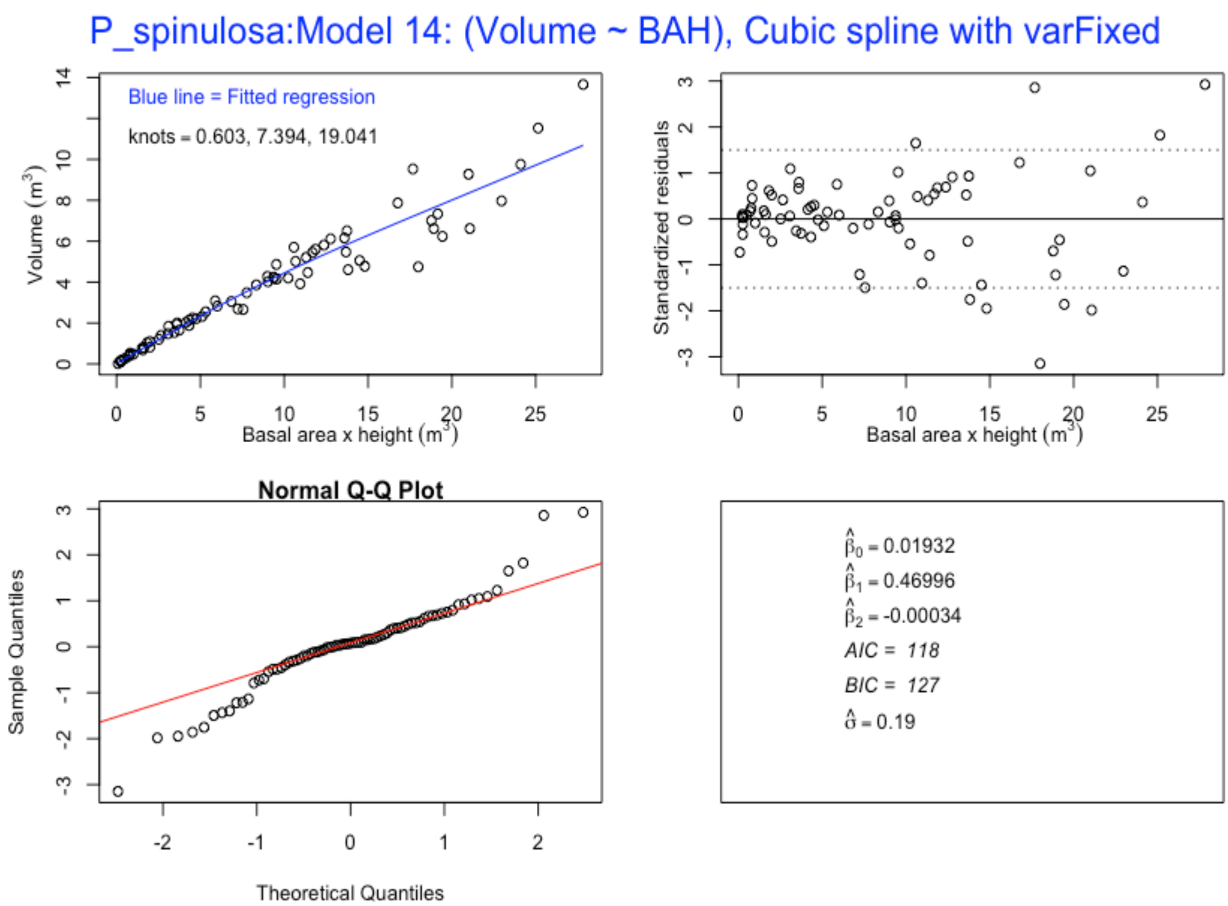
```
> ps.m14 <- gls(Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints,
              na.action=na.omit, weights = varFixed(~BAH.m3))
> summary(ps.m14)
```

```
Generalized least squares fit by REML
  Model: Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints
  Data: NULL
       AIC      BIC    logLik
  117.7171 126.8789 -54.85853

Variance function:
 Structure: fixed weights
 Formula: ~BAH.m3

Coefficients:
                         Value   Std.Error   t-value p-value
(Intercept)          0.0193226 0.030648492  0.630458  0.5304
BAH.m3               0.4699572 0.017633842 26.650865  0.0000
BAH.m3.splinepoints -0.0003418 0.000105674 -3.234859  0.0018
```

Plot of Model 14



6.15 Model 15– Volume with basal area * height (BAH) as predictor, with varPower

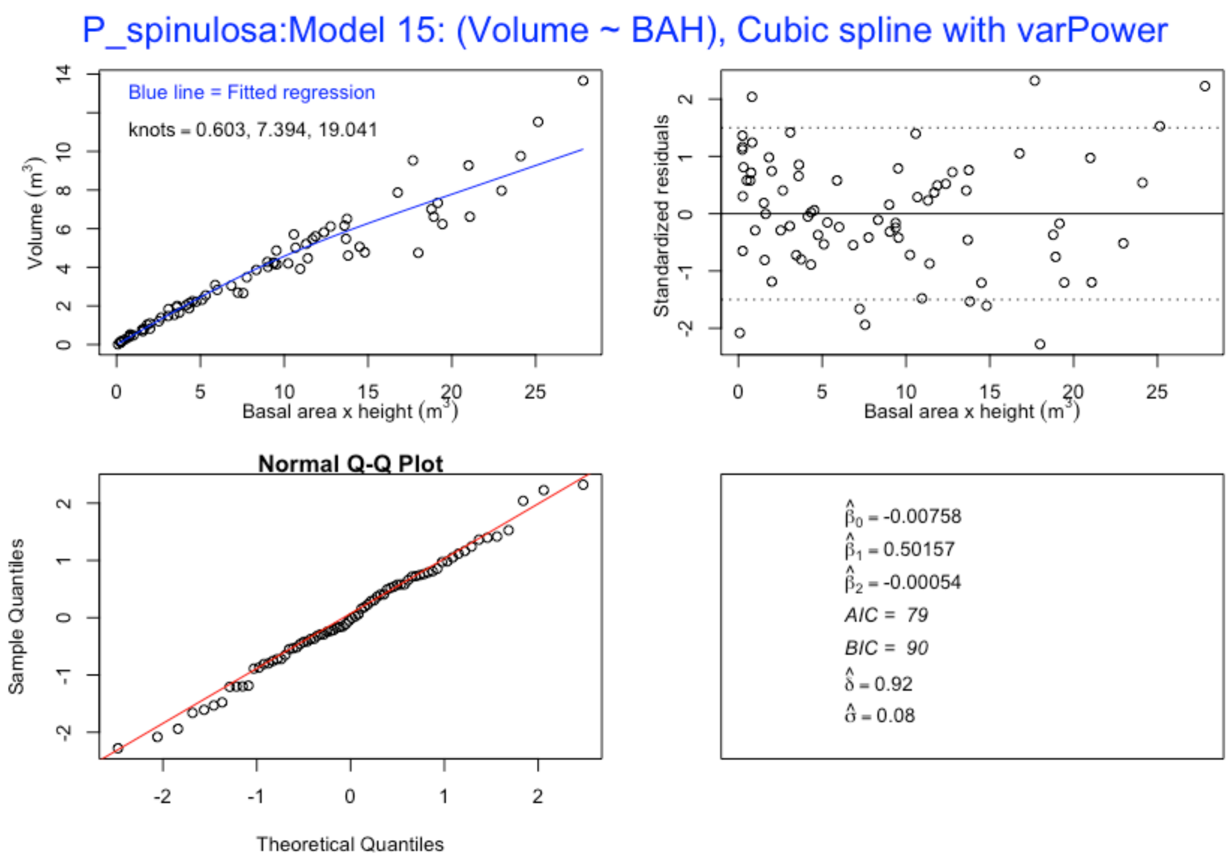
```
> ps.m15 <- gls(Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints,
              na.action=na.omit, weights = varPower(form =
              ~BAH.m3))
> summary(ps.m15)

Generalized least squares fit by REML
  Model: Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints
  Data: NULL
       AIC      BIC    logLik
  78.63281 90.08511 -34.3164

Variance function:
 Structure: Power of variance covariate
 Formula: ~BAH.m3
 Parameter estimates:
    power
0.9158461

Coefficients:
                         Value   Std.Error  t-value p-value
(Intercept)         -0.0075769 0.006419345 -1.18033  0.2417
BAH.m3               0.5015691 0.012545385 39.98036  0.0000
BAH.m3.splinepoints -0.0005436 0.000100758 -5.39475  0.0000
```

Plot of Model 15



6.16 Model 16 – Volume with basal area * height (BAH) as predictor, with varConstPower

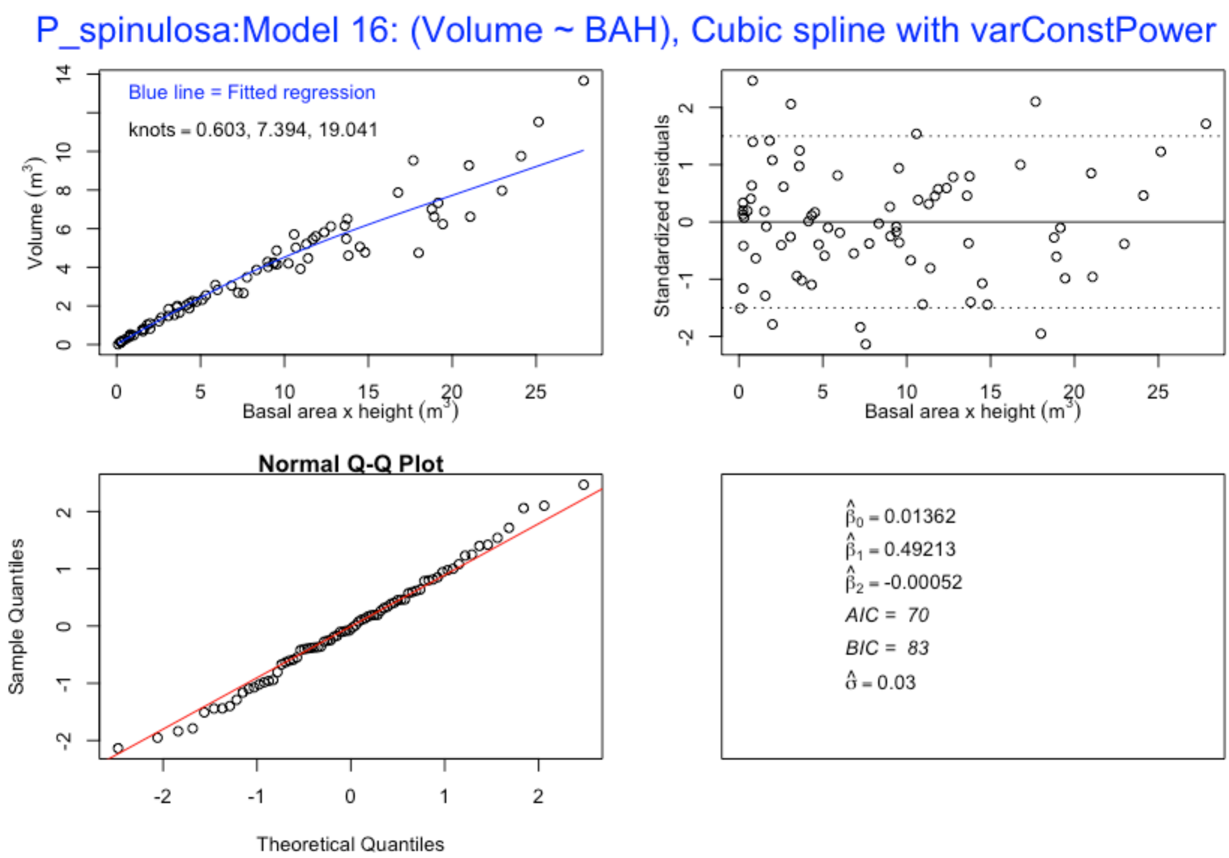
```
> ps.m16 <- gls(Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints,
            na.action=na.omit, weights = varConstPower(form =
            ~BAH.m3))
> summary(ps.m16)
```

```
Generalized least squares fit by REML
  Model: Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints
  Data: NULL
       AIC      BIC    logLik
  69.53446 83.27722 -28.76723

Variance function:
 Structure: Constant plus power of variance covariate
 Formula: ~BAH.m3
 Parameter estimates:
   const    power
0.760033 1.271253

Coefficients:
                         Value   Std.Error  t-value p-value
(Intercept)          0.0136202 0.010590769  1.28604  0.2025
BAH.m3               0.4921267 0.010986449 44.79397  0.0000
BAH.m3.splinepoints -0.0005172 0.000103892 -4.97869  0.0000
```

Plot of Model 16



2. Model evaluation using AIC and BIC values

SN	Model	AIC	BIC
1	Model 1 `> ps.m1 <- gls(Volume.m3 ~ DBH.cm)`	267	274
2	Model 2 `> ps.m2 <- gls(Volume.m3 ~ DBH.cm + DBH.cm.splinepoints, na.action=na.omit, weights = varFixed(~DBH.cm))`	243	252
3	Model 3 `> ps.m3 <- gls(Volume.m3 ~ DBH.cm + DBH.cm.splinepoints, na.action=na.omit, weights = varPower(form = ~DBH.cm))`	187	199
4	Model 4 `> ps.m4 <- gls(Volume.m3 ~ DBH.cm + DBH.cm.splinepoints, na.action=na.omit, weights = varConstPower(form = ~DBH.cm))`	178	192
5	Model 5 `> ps.m5 <- gls(Volume.m3 ~ BA.m2)`	244	250
6	Model 6 `> ps.m6<- gls(Volume.m3 ~ BA.m2 + BA.m2.splinepoints, na.action=na.omit, weights = varFixed(~BA.m2))`	172	181
7	Model 7 `> ps.m7 <- gls(Volume.m3 ~ BA.m2 + BA.m2.splinepoints, na.action=na.omit, weights = varPower(form = ~BA.m2))`	120	131
8	Model 8 `> ps.m8 <- gls(Volume.m3 ~ BA.m2 + BA.m2.splinepoints, na.action=na.omit, weights = varConstPower(form = ~BA.m2))`	119	133
9	Model 9 `> ps.m9 <- gls(Volume.m3 ~ DBH2H.m3)`	191	198
10	Model 10 `> ps.m10 <-gls(Volume.m3 ~ DBH2H.m3 + DBH2H.m3.splinepoints, na.action=na.omit, weights = varFixed(~DBH2H.m3))`	120	129

11	Model 11
	`> ps.m11 <-gls(Volume.m3 ~ DBH2H.m3 + DBH2H.m3.splinepoints,`
	`na.action=na.omit, weights = varPower(form = ~DBH2H.m3))`
		81	92

12	Model 12
	`> ps.m12 <- gls(Volume.m3 ~ DBH2H.m3 + DBH2H.m3.splinepoints,`
	`na.action=na.omit, weights = varConstPower(form = ~DBH2H.m3))`
		71	85

13	Model 13 `> ps.m13 <- gls(Volume.m3 ~ BAH.m3)`	190	197

14	Model 14
	`> ps.m14 <- gls(Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints,`
	`na.action=na.omit, weights = varFixed(~BAH.m3))`
		118	127

15	Model 15
	`> ps.m15 <- gls(Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints,`
	`na.action=na.omit, weights = varPower(form = ~BAH.m3))`
		79	90

16	Model 16
	`> ps.m16 <- gls(Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints,`
	`na.action=na.omit, weights = varConstPower(form = ~BAH.m3))`
		70	83

7. Selected Models

The best fitting models have been selected based on Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) values of the fitted models. The BIC value was mainly relied upon as it imposes a stronger penalty for the number of parameters in the model that need to be estimated. Smaller the values of AIC and BIC, better the fit of the model. Therefore, for *Picea spinulosa*, the selected models are;

1. Model 7 (Model which doesn't use height)
```
ps.m7 <- gls(Volume.m3 ~ BA.m2 + BA.m2.splinepoints,
           na.action=na.omit, weights =   varPower(form = ~BA.m2))
```

2. Model 16 (Model which uses the height)
```
ps.m16 <- gls(Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints,
            na.action=na.omit, weights = varConstPower(form = ~BAH.m3))
```

Two models have been selected for *Picea spinulosa*, one without height (X_1= BA which is model 7) and one with the height (X_1 = BAH, which is Model 16) as predictor or explanatory variable. Both the models have been fitted with natural (restricted) cubic spline function within a linear model framework. Although, nonlinear models are more flexible, they are more complicated than the linear models. The complications involved and amount of time and efforts spent on fitting nonlinear models often fail to justify by the improvements in the models. Moreover, the models fitted with natural (restricted) cubic spline functions perform well and track the curvilinearity better than nonlinear functions that were examined.

8. Demonstration of use of the selected best fit model

In general, the natural spline predictor with knots represented by t1, t2 and t3 takes the following form;

$$Y = \beta_0 + \beta_1 X + \beta_2 X_s + \varepsilon \qquad (7)$$

Where X_S corresponds to value in X as follows:

$$\mathrm{Xs} = \mathrm{g(X)} = (X - t1)_+^3 - (X - t2)_+^3 \frac{(t3-t1)}{(t3-t2)} + (X - t3)_+^3 \frac{(t2-t1)}{(t3-t2)} \qquad (8)$$

and the value of the positive part functions depend on the values of the knots as follows;

$$(X - t1)_+^3 = (X - t1)_+^3, \text{ if X > t1 and } (X - t1)_+^3{=}0, \text{ if X<t1} \qquad (9)$$

$$(X - t2)_+^3 = (X - t2)_+^3, \text{ if X> t2, and } (X - t2)_+^3 = 0, \text{ if X < t2} \qquad (10)$$

$$(X - t3)_+^3 = (X - t3)_+^3, \text{ if X > t3, and } (X - t3)_+^3{=}0, \text{ if X<t3} \qquad (11)$$

Where t1, t2 and t3 for the above models are 10^{th}, 50^{th} and 90^{th} percentiles and are called knots. The values of knots differ from species and models.

To demonstrate use of the selected models for *Picea spinulosa* – model 7, the knots t1, t2 and t3 are 0.033, 0.235 and 0.449 as generated by the model. The model 7 has been fitted with volume as function of basal area in meter square (BA) i.e

$$BA = \pi r^2 \tag{12}$$

where in

$$r^2 = \left[\frac{dbh}{2*100}\right]^2 \tag{13}$$

Where r is radius in meters and dbh is diameter at breast height in centimeters.

Therefore, *Picea spinulosa* with diameter of 59 cm resulting in basal area of 0.273397101 m^2, the volume can be estimated using the above equation (model 7) as below. But first the value of BA.m2 has to be calculated, which is;

$$\begin{aligned}
\text{BA} \quad &= \pi r^2 = \frac{\pi*59^2}{200^2} = 0.273397101\ \text{m}^2 \\
\text{g(X)} \quad &= (X - t1)_+^3 - (X - t2)_+^3 \frac{(t3-t1)}{(t3-t2)} + (X - t3)_+^3 \frac{(t2-t1)}{(t3-t2)}
\end{aligned}$$

$$\begin{aligned}
\text{g(BA)} \quad &= (BA - t1)_+^3 - (BA - t2)_+^3 \frac{(t3-t1)}{(t3-t2)} + (BA - t3)_+^3 \frac{(t2-t1)}{(t3-t2)} \\
\text{g(BA)} \quad &= (0.0.273397101 - 0.033)_+^3 - (0.273397101 - 0.235)_+^3 \frac{(0.449-0.033)}{(0.449-0.235)} + 0 \\
&= (0.240397101)_+^3 - (0.038397101)_+^3 \frac{(0.416)}{(0.214)} + 0 \\
&= (0.240397101)_+^3 - (0.038397101)_+^3 * 1.943925234 + 0 \\
&= 0.013892733 - 0.00005661028*1.943925234 \\
&= 0.013892733 - 0.00011005 \\
&= 0.01378269
\end{aligned}$$

Hence, the volume predicted for this tree by the selected model (model 7) is

$$\begin{aligned}
\text{V} &= \beta_0 + \beta_1 . BA + \beta_2 BA.m_2 + \varepsilon \\
&= -0.103695 + 13.549084 * 0.273397101 + 17.585685 * 0.01124419 \\
&= -0.103695 + 3.7042803 + 0.24237798 \\
&= \mathbf{3.842963\ m^3}
\end{aligned}$$

Similarly, to demonstrate model 16 with t1, t2 and t3 of 0.603, 7.394 and 19.041 respectively, we considered this same tree but with height, i.e dbh = 59 cm resulting in BA = 0.273397101 m^2 and height (H) = 34.86 m.

$$\begin{aligned}
\text{BAH} &= 0.273397101 \text{ x } 34.86 \\
&= 9.53062294086
\end{aligned}$$

$$\text{g(X)} \quad = (X - t1)_+^3 - (X - t2)_+^3 \frac{(t3-t1)}{(t3-t2)} + (X - t3)_+^3 \frac{(t2-t1)}{(t3-t2)}$$

$$\text{g(BAH)} = (BAH - t1)_+^3 - (BAH - t2)_+^3 \frac{(t3-t1)}{(t3-t2)} + (BAH - t3)_+^3 \frac{(t2-t1)}{(t3-t2)}$$

$$= (9.53062294086 - 0.603)_+^3 - (9.53062294086 - 7.394)_+^3 \frac{(19.041-0.603)}{(19.041-7.394)} + 0$$
$$= (8.92762293)_+^3 - (2.13662294086)_+^3 \frac{(18.438)}{(11.647)} + 0$$
$$= 711.5534307 - 9.754020438 * 1.5830686 + 0$$
$$= 711.5534307 - 15.44128347 + 0$$
$$= 696.112147$$

Hence, the volume predicted by model 15 for this tree is;

$$\text{V} = \beta_0 + \beta_1 . BAH.m3 + \beta_2 BAH.m3_2 + \varepsilon$$

$$= 0.0136202 + 0.4921267 * 9.53062294086 + (-0.0005172 * 696.112147)$$

$$= 0.0136202 + 4.690274 + (-0.3600292)$$

= **4.343865 m^3**

The field measured volume for this particular tree with DBH of 59 cm and height of 34.86 m is 4.666577491 m^3.

9. Model Performance

To assess the performance of selected models, we compared the volume predicted by selected models (7 and 16) with the volume of the tree as measured in the field. Using the equations of the selected models, volume prediction or estimation was done in R.

SN	Tree_ ID	Height (in m)	DBH (in cm)	Volume in m^3 (Field measured) [A]	Predicted Volume Model_7 [B]	Predicted Volume Model_16 [C]	Difference (Field – Model_7) [A – B]	Difference (Field – Model_16) [A – C]
1	psec01	34.86	59	4.8661657	3.842963265	4.343865009	1.023202435	0.522300691
2	psec02	20.44	35.2	1.094367066	1.219498371	0.991130916	-0.125131305	0.10323615
3	psec05	15.62	20.2	0.26684223	0.330517734	0.259969072	-0.063675504	0.006873158
4	psec06	20.08	40	1.20273919	1.612925384	1.251754342	-0.410186194	-0.049015152
5	psec07	36.15	52.3	3.482030147	2.912761783	3.64546608	0.569268364	-0.163435933
6	psec08	25.95	27.3	0.774571595	0.68969261	0.760754161	0.084878985	0.013817434
7	psec09	25.2	22.5	0.472797599	0.43503261	0.506684383	0.037764989	-0.033886785
8	psec10	30.8	42.3	1.87767627	1.822229904	2.11697203	0.055446366	-0.239295761
9	psec11	14.18	16.5	0.165102763	0.186017814	0.162834666	-0.020915052	0.002268097
10	psec12	28.3	39.4	1.53984739	1.560602183	1.699711568	-0.020754793	-0.159864177
11	psec13	14.87	15	0.130968537	0.135737078	0.142938592	-0.004768541	-0.011970055
12	psec14	22.41	20.8	0.41662656	0.356695658	0.3883627	0.059930902	0.028263859
13	psec16	14.97	14.8	0.107362349	0.129394788	0.140359715	-0.022032439	-0.032997366
14	psec18	19.8	21.3	0.378038843	0.379096163	0.360828828	-0.00105732	0.017210014
15	psec19	22.8	29.6	0.686314748	0.829471942	0.78527455	-0.143157193	-0.098959802
16	psec20	28.35	46.2	2.220080923	2.210574218	2.315523966	0.009506705	-0.095443044
17	psec21	10.7	10	0.01824963	0.002719257	0.054977348	0.015530373	-0.036727719
18	psec22	23.8	29.5	0.807256605	0.823151856	0.813614105	-0.01589525	-0.0063575
19	psec24	28.16	49	2.545480691	2.517528563	2.572988334	0.027952128	-0.027507643
20	psec25	30.85	65	4.201513264	4.830553676	4.607862861	-0.629040412	-0.406349597
21	psec26	39.7	75.3	9.531324031	6.844799165	7.029628181	2.686524866	2.50169585
22	psec27	32.8	44.5	2.321451544	2.035923028	2.477045278	0.285528516	-0.155593734
23	psec28	65.2	39.8	2.032063803	1.59538148	3.786890472	0.436682323	-1.754826669
24	psec29	61.55	40	1.999646078	1.612925384	3.632461969	0.386720694	-1.632815891
25	psec30	81.2	55.3	3.863770369	3.306935895	7.573438755	0.556834474	-3.709668386
26	psec31	35.7	45.8	3.084966579	2.168708563	2.832003259	0.916258016	0.25296332
27	psec32	38.15	54.8	4.276422056	3.238771397	4.139152072	1.037650659	0.137269984
28	psw01	31.9	42.5	2.247760066	1.841100437	2.209485988	0.406659629	0.038274078
29	psw02	32.8	69.3	5.812673044	5.625607107	5.360022166	0.187065937	0.452650878
30	psw03	46.5	87.3	13.65888883	9.600539824	10.05479061	4.058349006	3.604098218
31	psw04	34.3	59	4.191450669	3.842963265	4.285533298	0.348487404	-0.094082629
32	psw05	19.8	23	0.483692927	0.459247401	0.418458687	0.024445526	0.06523424
33	psw06	12.9	15.5	0.142818451	0.151965252	0.133410021	-0.009146801	0.00940843
34	psw08	23.4	38	1.393831431	1.442070395	1.315179147	-0.048238965	0.078652283

35	pswc01	48.8	81	11.52829878	8.103058718	9.254446618	3.425240062	2.273852161
36	pswc02	10.4	17	0.133388215	0.203842202	0.129791271	-0.070453988	0.003596944
37	pswc03	27.3	38	1.83920336	1.442070395	1.529295466	0.397132965	0.309907894
38	pswc04	43.4	63.5	6.508100701	4.570071308	5.81353051	1.938029393	0.694570191
39	pswc05	25.5	46.5	2.142622231	2.242290956	2.117978914	-0.099668725	0.024643318
40	pswc07	45.7	57.5	5.604516509	3.618924513	5.187813826	1.985591995	0.416702682
41	pswc08	14.2	27	0.528043333	0.672315885	0.413728427	-0.144272552	0.114314907
42	psw09	22	32.5	1.035949449	1.022498222	0.910841433	0.013451227	0.125108017
43	psw10	40	85.5	7.970598003	9.161248193	8.604901958	-1.19065019	-0.634303955
44	psw11	32	48.9	2.829294689	2.506133641	2.889439716	0.323161048	-0.060145027
45	psw12	44	74	6.616100754	6.570791249	7.400964918	0.045309505	-0.784864164
46	psw13	35	65.2	5.447341087	4.865956832	5.125134649	0.581384255	0.322206438
47	psw14	35	36.9	1.650496634	1.352361158	1.839600688	0.298135476	-0.189104054
48	psw15	19.3	45	1.477928082	2.08640782	1.516458014	-0.608479737	-0.038529931
49	psw16	24.4	70.6	4.151098905	5.879480731	4.351933977	-1.728381826	-0.200835073
50	psw17	20.7	47	1.939720731	2.295764147	1.767210002	-0.356043416	0.172510729
51	psw18	19.4	36.2	0.823139373	1.296811664	0.994840316	-0.473672291	-0.171700943
52	psw19	36.3	69.6	4.60898949	5.683656182	5.834919207	-1.074666692	-1.225929717
53	psw20	28.7	57.9	2.662923413	3.677769411	3.558551067	-1.014845998	-0.895627655
54	psw21	30	67	5.702920458	5.191594252	4.732056321	0.511326206	0.970864137
55	psw22	38	55	4.020903372	3.265915776	4.150879403	0.754987596	-0.129976031
56	psw23	30.5	69	4.469205615	5.567884893	5.027206461	-1.098679278	-0.558000846
57	psw24	31	54.5	2.690652089	3.198357767	3.421921207	-0.507705678	-0.731269118
58	psw25	33	65	3.918821309	4.830553676	4.866441543	-0.911732367	-0.947620234
59	psw26	42	67	4.774218913	5.191594252	6.152162362	-0.417375339	-1.377943449
60	psw27	43.7	65	5.056709733	4.830553676	6.055459567	0.226156057	-0.998749834
61	psw28	44	75	6.234599201	6.781122541	7.554328947	-0.54652334	-1.319729746
62	psw29	39	74	7.871302187	6.570791249	6.756952298	1.300510938	1.114349889
63	psw30	49	74	6.618694069	6.570791249	8.04146171	0.04790282	-1.42276764
64	psw31	56	66	7.334946347	5.009134635	7.47095682	2.325811712	-0.136010473
65	psw32	41.5	86	9.755593634	9.282355631	8.944659749	0.473238003	0.810933884
66	psw33	41.4	76	7.008484823	6.994378386	7.358441252	0.014106437	-0.34995643
67	psw34	40	66	5.472266154	5.009134635	5.794228783	0.463131519	-0.321962629
68	psw35	43	73	4.747888249	6.36353533	7.124656937	-1.615647081	-2.376768688
69	psw36	10.2	17.5	0.139818007	0.222198662	0.13435795	-0.082380655	0.005460057
70	psw37	24	82.3	6.117037503	8.4028826	5.492853005	-2.285845097	0.624184498
71	psw38	27	99.5	9.273938156	12.81918712	8.01766533	-3.545248965	1.256272825
72	psw39	26.3	74	5.209930115	6.570791249	4.994330898	-1.360861134	0.215599217
73	psw40	25.3	68.7	4.239919873	5.510491975	4.285829279	-1.270572102	-0.045909406
74	psw41	23.4	86	6.158016209	9.282355631	5.764341165	-3.124339421	0.393675044
75	psw42	24.7	74.2	5.01539423	6.612616174	4.769552519	-1.597221944	0.245841711
76	psw43	22.1	62.8	3.051576188	4.451583439	3.256632624	-1.400007252	-0.205056437
				276.6753821	**276.3753615**	**282.5781302**	**0.300020672**	**-5.902748092**

From the above table, the difference [A-B] provides difference between the volume measured in the field (actual volume) and the volume predicted by model 7. The figures with negative (-) indicates that the volume has been over-predicted by the model 7 vis-à-vis actual volume of the particular tree. And the figures without negative (-) sign indicates the under prediction of volume by the model 7.

Similarly, the difference [A-C] is the difference between the actual volume and the volume predicted by the model 16. Same explanation is applicable here – the figures with negative (-) sign indicates overprediction of volume by the model, while those figures without negative (-) are under prediction of volume by the model 16.

Summation of the figures in the difference column result in 0.300020672 and -5.902748092 for model 7 and model 16 respectively. These indicate that the model 7 under predicts total volume for 76 trees by 0.300020672 m^3, while the model 16 overpredicts by 5.902748092 m^3 of volume for 76 trees.

10. Limitations of the model

The model has the following limitations;

1. The modeling has been done based on only 76 sample trees. The model can further be improved by increasing the samples.

2. The diameter for the sample trees ranges between minimum of 10 cm to 99.5 cm (over bark). However, the model prediction for trees of bigger diameter classes should be done with caution, since there were lesser number of bigger diameter trees in the sample.

11. Conclusion

The model 7 which doesn't use the height performs better than the model 16 that use the height as predictor, as empirically shown above. Unlike, in other conifer species that we have modelled (*Pinus wallichiana, Juniperus recurva, Tsuga dumosa*) for which the model with height performed much better, but in *Picea spinulosa*, model with height seems to be performing not so well. Nonetheless, since AIC and BIC values are much lesser for model 16 vis-à-vis model 7.

But since the two models are fitted with different predictors (one with and other without height as predictor), it leads us to propose two best fit models for *Picea spinulosa*, namely;

1. Model 7: the best fit model that doesn't use height
2. Model 16: the best fit model which uses height as predictor.

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14. Annexure – Dataset for *Picea spinulosa*

SN	Tree_ID	Height.m	DBH.cm	Volume.m3	BA.m2	BAH.m3	DBH2H.m3
1	psec01	34.86	59	4.866166	0.273397	9.530623	12.13477
2	psec02	20.44	35.2	1.094367	0.097314	1.989098	2.532598
3	psec05	15.62	20.2	0.266842	0.032047	0.50058	0.637358
4	psec06	20.08	40	1.202739	0.125664	2.523327	3.2128
5	psec07	36.15	52.3	3.48203	0.214829	7.766075	9.888073
6	psec08	25.95	27.3	0.774572	0.058535	1.518982	1.934028
7	psec09	25.2	22.5	0.472798	0.039761	1.001972	1.27575
8	psec10	30.8	42.3	1.877676	0.140531	4.32834	5.511013
9	psec11	14.18	16.5	0.165103	0.021382	0.303203	0.386051
10	psec12	28.3	39.4	1.539847	0.121922	3.450395	4.393179
11	psec13	14.87	15	0.130969	0.017671	0.262775	0.334575
12	psec14	22.41	20.8	0.416627	0.033979	0.76148	0.969546
13	psec16	14.97	14.8	0.107362	0.017203	0.257534	0.327903
14	psec18	19.8	21.3	0.378039	0.035633	0.705528	0.898306
15	psec19	22.8	29.6	0.686315	0.068813	1.568947	1.997645
16	psec20	28.35	46.2	2.220081	0.167639	4.752552	6.051137
17	psec21	10.7	10	0.01825	0.007854	0.084038	0.107
18	psec22	23.8	29.5	0.807257	0.068349	1.626713	2.071195
19	psec24	28.16	49	2.545481	0.188574	5.310247	6.761216
20	psec25	30.85	65	4.201513	0.331831	10.23698	13.03413
21	psec26	39.7	75.3	9.531324	0.445328	17.67951	22.51026
22	psec27	32.8	44.5	2.321452	0.155528	5.101334	6.49522
23	psec28	33.3	39.8	2.032064	0.12441	4.14286	5.274853
24	psec29	28.75	40	1.999646	0.125664	3.612832	4.6
25	psec30	34.7	55.3	3.86377	0.240182	8.334309	10.61157

26	psec31	35.7	45.8	3.084967	0.164748	5.881513	7.488575
27	psec32	38.15	54.8	4.276422	0.235858	8.997991	11.4566
28	psw01	31.9	42.5	2.24776	0.141863	4.525415	5.761938
29	psw02	32.8	69.3	5.812673	0.377187	12.37172	15.75217
30	psw03	46.5	87.3	13.65889	0.598575	27.83372	35.439
31	psw04	34.3	59	4.191451	0.273397	9.377521	11.93983
32	psw05	19.8	23	0.483693	0.041548	0.822642	1.04742
33	psw06	12.9	15.5	0.142818	0.018869	0.243413	0.309923
34	psw08	23.4	38	1.393831	0.113411	2.653829	3.37896
35	pswc01	48.8	81	11.5283	0.5153	25.14663	32.01768
36	pswc02	10.4	17	0.133388	0.022698	0.236059	0.30056
37	pswc03	27.3	38	1.839203	0.113411	3.096134	3.94212
38	pswc04	43.4	63.5	6.508101	0.316692	13.74444	17.49997
39	pswc05	25.5	46.5	2.142622	0.169823	4.330479	5.513738
40	pswc07	45.7	57.5	5.604517	0.259672	11.86702	15.10956
41	pswc08	14.2	27	0.528043	0.057256	0.813028	1.03518
42	psw09	22	32.5	1.035949	0.082958	1.825069	2.32375
43	psw10	40	85.5	7.970598	0.574146	22.96583	29.241
44	psw11	32	48.9	2.829295	0.187805	6.009766	7.651872
45	psw12	44	74	6.616101	0.430084	18.9237	24.0944
46	psw13	35	65.2	5.447341	0.333876	11.68566	14.87864
47	psw14	35	36.9	1.650497	0.106941	3.742921	4.765635
48	psw15	19.3	45	1.477928	0.159043	3.069532	3.90825
49	psw16	24.4	70.6	4.151099	0.391471	9.551886	12.16184
50	psw17	20.7	47	1.939721	0.173494	3.591335	4.57263
51	psw18	19.4	36.2	0.823139	0.102922	1.996681	2.542254
52	psw19	36.3	69.6	4.608989	0.380459	13.81068	17.5843
53	psw20	28.7	57.9	2.662923	0.263298	7.556643	9.621417
54	psw21	30	67	5.70292	0.352565	10.57696	13.467
55	psw22	38	55	4.020903	0.237583	9.028152	11.495
56	psw23	30.5	69	4.469206	0.373928	11.40481	14.52105
57	psw24	31	54.5	2.690652	0.233283	7.23177	9.207775
58	psw25	33	65	3.918821	0.331831	10.95041	13.9425
59	psw26	42	67	4.774219	0.352565	14.80774	18.8538
60	psw27	43.7	65	5.05671	0.331831	14.501	18.46325
61	psw28	44	75	6.234599	0.441786	19.4386	24.75
62	psw29	39	74	7.871302	0.430084	16.77328	21.3564
63	psw30	49	74	6.618694	0.430084	21.07412	26.8324
64	psw31	56	66	7.334946	0.342119	19.15869	24.3936
65	psw32	41.5	86	9.755594	0.58088	24.10654	30.6934
66	psw33	41.4	76	7.008485	0.453646	18.78094	23.91264
67	psw34	40	66	5.472266	0.342119	13.68478	17.424

68	psw35	43	73	4.747888	0.418539	17.99716	22.9147
69	psw36	10.2	17.5	0.139818	0.024053	0.245339	0.312375
70	psw37	24	82.3	6.117038	0.531973	12.76735	16.2559
71	psw38	27	99.5	9.273938	0.777564	20.99422	26.73068
72	psw39	26.3	74	5.20993	0.430084	11.31121	14.40188
73	psw40	25.3	68.7	4.23992	0.370684	9.378295	11.94082
74	psw41	23.4	86	6.158016	0.58088	13.5926	17.30664
75	psw42	24.7	74.2	5.015394	0.432412	10.68058	13.59893
76	psw43	22.1	62.8	3.051576	0.309748	6.845441	8.715886