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ཚལ་ལྷན་ཁག། རགས་ཚལ་དང་སྤོང་ཀ་འབྲས་
རྟོག་ལས་ཁུངས།



Royal Government of Bhutan
Ministry of Agriculture and Forests
Department of Forests and Park Services

SPECIES SPECIFIC VOLUME EQUATION TO ESTIMATE MERCHANTABLE VOLUME

Abies densa

Forest Resources Management Division
Department of Forest and Park Services
Ministry of Agriculture and Forests

2018

Species specific volume equation to
estimate merchantable volume

Abies densa

December, 2018

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1. Summary

The volume equation developed in this study will predict the merchantable volume of *Abies densa*. The merchantability standard adopted for this study are the trees above 10 cm in diameter at breast height (dbh) and top diameter measured up to 10 cm over bark have been considered for volume calculation.

A total of 16 models were fitted. First 4 models were fitted with volume as a function of diameter at breast height (DBH), while models 5 – 8 were fitted with basal area (BA) as the predictor variable. With product of squared diameter at breast height and height (DBH2H) as predictor variable, 4 models, namely the models 9 – 12 were fitted. The last four models, 13 -16 were fitted with product of basal area and height (BAH) as the predictor.

The initial plots of response variables (volume) and predictor (DBH, BA, DBH2H and BAH) variables clearly indicated presence of heteroscedasticity, which has been modeled using variance functions (varFixed, varPower and varConstPower) in gls () function of nlme package.

Of the sixteen, two models viz model 7 (basal area as predictor) for those models which were fitted without height and model 15 (basal area x height as predictor) for those models which were fitted with height as predictors, have been selected as the best fit models. The model 7 had AIC and BIC values of 35 and 44 respectively, whereas the model 15 had AIC and BIC values of 23 and 32 respectively. Lower the AIC and BIC values, better the fit of the model.

The performance of the selected models was assessed by comparing the actual volume with the volumes predicted by two selected models for each tree.

2. Introduction

The volume equations, developed during pre-investment survey (PIS) carried out between 1974-81 predict total tree volume, and not the merchantable volume of trees. The recent change of policy of the Department of Forests and Park Services to allot timber for rural house construction in the form of log volume instead of allotting by number of trees as was once practiced, has necessitated development of merchantable log volume equation.

Therefore, standards of merchantability adopted for this study to develop merchantable log volume equation are, all trees above 10 cm diameter at breast height (dbh) and the sections up to 10 cm top diameter over the bark.

As was done for PIS exercise to develop volume equation, this study ignores/does not consider the volume of foliage and branches for the purpose of calculating the merchantable volume. This decision stems from the objective, which is to estimate merchantable log volume. Moreover, branches are rarely used as timber (at least in Bhutan) and are mostly used for firewood.

The sample trees for this study have been felled as part of biomass equation development field work. The data protocol for biomass equation development required collecting a minimum of 8 trees each from four regions of Bhutan namely, eastern, eastern central, western and western central. Therefore, 46 trees in total have been felled for *Abies densa* from four regions namely; eastern, east-central, western-central and western regions.

The trees were felled at 0.3 m height from the ground at which the diameter was measured and recorded. Then diameter at zero height (ground level) were also measured and recorded. After felling, the diameter was measured at 0.7 m from 0.3 m height (essentially making 1 m height, i.e 0.3 m + 0.7 m =1 m). Thereafter, at every meter length, the diameter was measured and recorded, thus making many 1 m length sections of log. As mentioned above the smallest top diameter considered for merchantable log volume calculation was up to 10 cm diameter over bark. Top sections below 10 cm diameter have been discarded.

3. Volume Calculation

Trees after felling are converted into different sizes of sections depending on the requirement and demand. Sections with length of 8 or more feet long are called logs and shorter ones are called sticks or bolts (Avery and Burkhardt, 1994). The scaling or measuring the volume of the section is done by multiplying the length with the cross-sectional area of the section. Although they rarely form true circles, they are assumed so for the purpose of calculating cross sectional area in meter square, which is

$$\text{Cross sectional area (A)} = A = \pi r^2 = \frac{\pi D^2}{4 \cdot 10000} \quad (1)$$

Where **r** is radius in meters and **D** is diameter at breast height in centimeters.

From the ground level to 0.3 m height (height at which sample tree has been cut) is section I, while 0.3 m to 0.7 m is section II. The subsequent sections of 1 m length each are numbered III, IV and so on. The last section is the terminal section, whose length is equal to or less than 1 m. As was adopted for PIS, in this study too the branch volumes are ignored assuming that rarely branches yield merchantable timber.

The most commonly used formulae for calculating volume are the Huber, Newton and Smalian's formulae (Sadiq, 2006, and Goulding, 1979). Of the three commonly used volume calculation approaches or formulae, we have used Smalian's formula to calculate volume (in m³) for this study, which is;

$$\text{Section volume } (V_s) = \frac{A+a}{2} * L \quad (2)$$

Where A = Cross sectional area in m² at large end of the section
 a = Cross sectional area in m² at small end of the section
 L = Length of the section in meter

Smalian's formula is the easiest and least expensive to apply and therefore applied to get volume for each section of the sample trees. However, for the terminal section, the following formula was used to calculate the volume, which is;

$$\text{Terminal section volume } (V_t) = \frac{A}{3} * L \quad (3)$$

The volume for sections and terminal section for individual trees were then summed to obtain the total volume for each individual sample tree, which is;

$$\text{Volume of tree } (V) = \sum_{s=1}^n V_s + V_t \quad (4)$$

After obtaining individual tree volume (Volume.m3), it was then tabulated against the variables - height in meter (Height.m) and the diameter at breast height in centimeter (DBH.cm) readings and thus stored in the excel file.

4. The Dataset used for modeling volume of *Abies densa*

A total of 46 trees have been felled and collected data from four regions for this study. The summary of data set is presented below, while the detailed dataset is presented as an annexure to this document.

4.1 Summary descriptive statistics of *Abies densa* dataset

```
> summary(ad)
```

Tree_ID	Height.m	DBH.cm	Volume.m3
ade01 : 1	Min. : 8.40	Min. :14.70	Min. :0.09395
ade02 : 1	1st Qu.:20.09	1st Qu.:29.80	1st Qu.:0.86882
ade03 : 1	Median :28.02	Median :39.05	Median :1.55011
ade04 : 1	Mean :29.47	Mean :41.95	Mean :2.51704
ade05 : 1	3rd Qu.:37.48	3rd Qu.:53.73	3rd Qu.:3.61125
ade06 : 1	Max. :66.80	Max. :82.00	Max. :9.91629

BA.m2	BAH.m3	DBH2H.m3
Min. :0.01697	Min. : 0.1585	Min. : 0.2018
1st Qu.:0.06978	1st Qu.: 1.3868	1st Qu.: 1.7658
Median :0.11978	Median : 3.0237	Median : 3.8499
Mean :0.16052	Mean : 5.7169	Mean : 7.2789
3rd Qu.:0.22675	3rd Qu.: 8.8553	3rd Qu.: 11.2749
Max. :0.52810	Max. : 20.8600	Max. : 26.5598

5. Fitting the models

The models have been fitted in R, which is a robust statistical computing environment. It is a powerful tool which provides wide range of statistical and graphical options to explore, calculate and manage data besides modelling. It is very powerful and widely used statistical tool which is free and allows user to customize the scripts depending on desired output, which is not possible in many of the statistical softwares.

After reading in the excel files into R, we created other variables namely; basal area in square meter (BA.m2), basal area in meter times height in meter (BAH.m3) and square of the diameter in meter times height in meter (DBH2H.m3). The height in meter (Height.m) and diameter in centimeter (DBH.cm) were measured and recorded in the field.

Prior to fitting models, we explored and examined each set of data by preparing descriptive summaries that provided mean, median and range of dependent/response and independent/predictor variables. Then we plotted scatter graphs which provided sense of relationship between the dependent/response (volume) and independent/predictor variables (namely DBH.cm, BA.m2, DBH2H.m3 and BAH.m3). These graphs showed curvilinear relationship between response and predictor variables. The scatter plots also clearly revealed the presence of phenomenon, referred in statistical parlance, as heteroscedasticity, which is the increase in variation in response (volume) variable with increase in value of the predictor variables.

Therefore, we fitted the models using the `gls ()` function of the `nlme` package of R, because the `gls ()` function has the capability to model heteroscedasticity. We didn't transform the variables, mainly response variable, because transformation makes it difficult to directly interpret the relationship between response and predictor variables; and secondly to compare the AIC and BIC values among the different models, the response variables need to be identical.

The models were fitted with volume as a function of four variables;

- 1) DBH.cm,
- 2) BA.m2,
- 3) DBH2H.m3 and
- 4) BAH.m3.

For each of the variable, we fitted one simple `gls ()` function, which can be written in the following form;

$$Y = \beta_0 + \beta_1 X + \epsilon, \quad (5)$$

Where $Y = \text{Volume (V)}$ and $X = \text{predictor variable}$

And then fitted 3 models with restricted natural cubic spline functions. The restricted natural cubic spline function enables better tracking of curvilinear relationship between response and predictor variables. These models introduce an additional predictor variable as part of a 3 knot-cubic spline. They take the following forms;

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \epsilon, \quad (6)$$

Where $Y = \text{Response variable, volume (V)}$

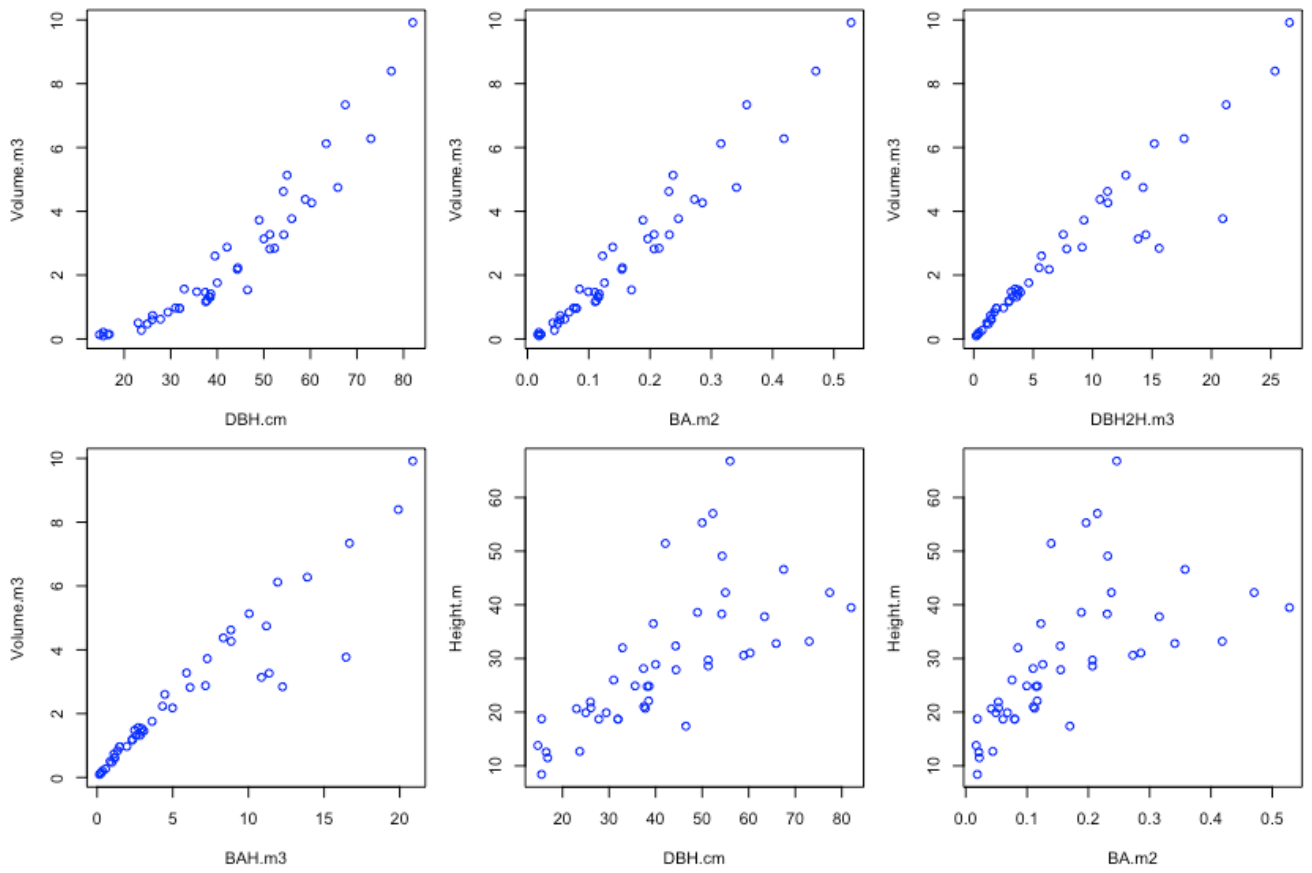
$X_1 = \text{Predictor variable}$

$X_2 = g(X_1)$

And $g(X_1)$ is the spline transformation of X_1 predictor variable

6. Summary Plots

Abies densa (N = 46)



7. Models and results

7.1. Model 1 - Volume with diameter at breast height (DBH) as predictor

```
> ad.m1 <- gls(Volume.m3 ~ DBH.cm)
> summary(ad.m1)
```

Generalized least squares fit by REML

Model: Volume.m3 ~ DBH.cm

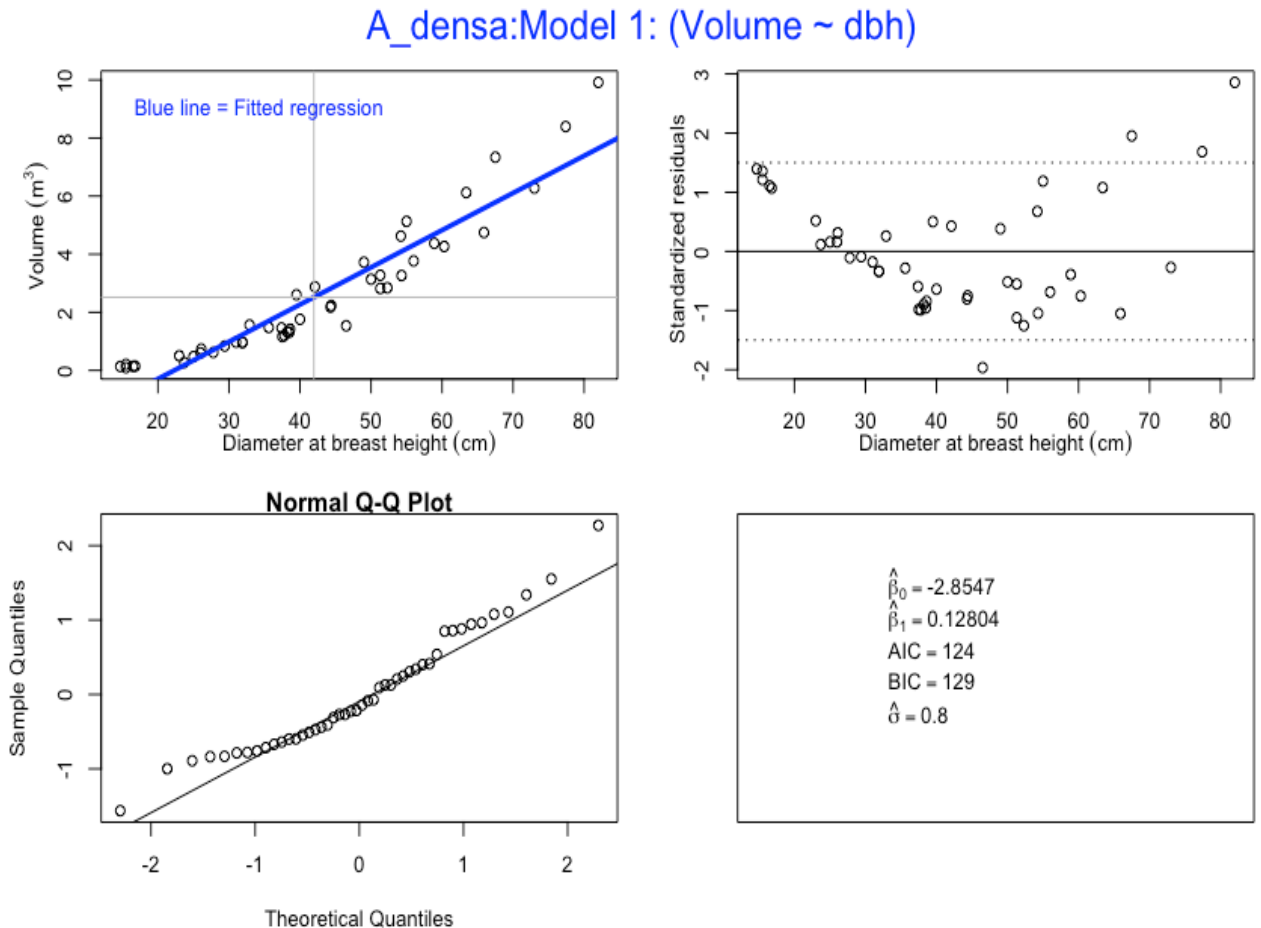
Data: NULL

	AIC	BIC	logLik
	123.9833	129.3359	-58.99166

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	-2.8547000	0.31464183	-9.072856	0
DBH.cm	0.1280378	0.00695977	18.396833	0

Plot of model 1



7.2. Model 2 - Volume with diameter at breast height (DBH) as predictor, with varFixed

```
> ad.m2 <- gls(Volume.m3 ~ DBH.cm + DBH.cm.splinepoints,
               na.action=na.omit, weights = varFixed(~DBH.cm))
```

```
> summary(ad.m2)
```

Generalized least squares fit by REML

Model: Volume.m3 ~ DBH.cm + DBH.cm.splinepoints

Data: NULL

	AIC	BIC	logLik
	98.78118	105.826	-45.39059

Variance function:

Structure: fixed weights

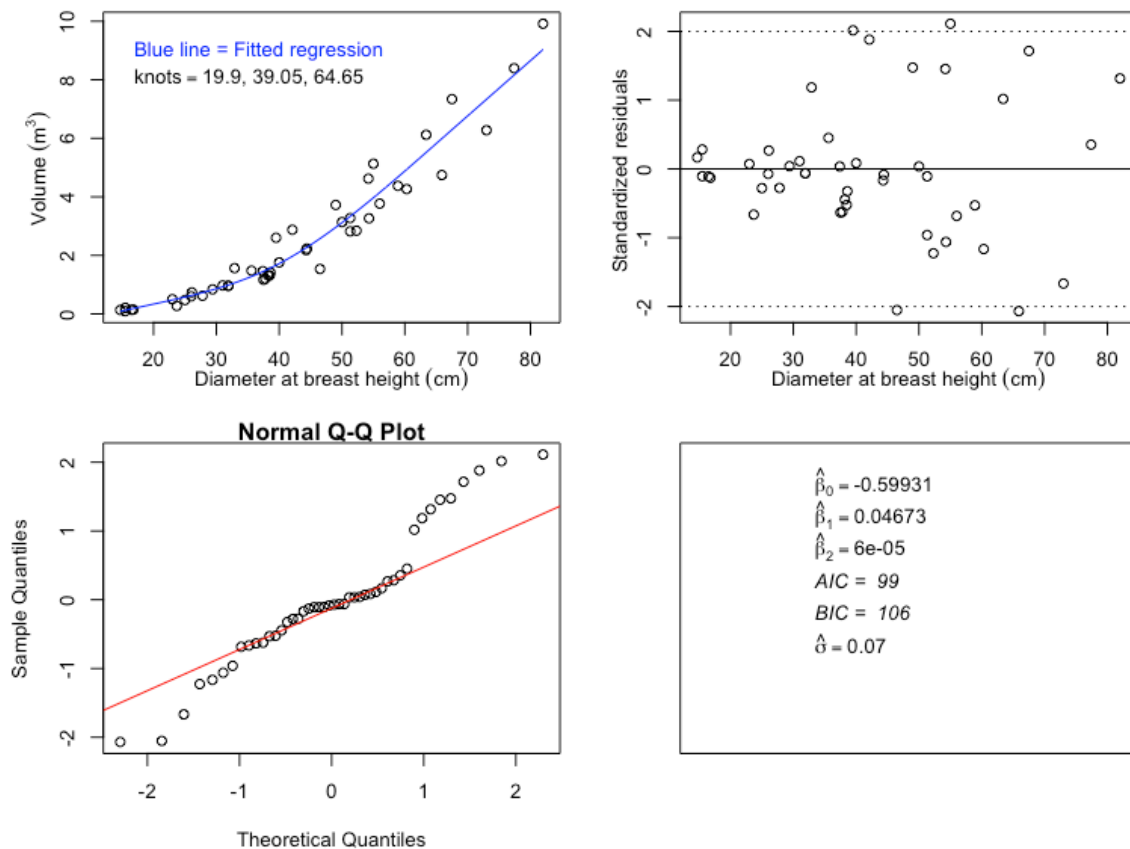
Formula: ~DBH.cm

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	-0.5993076	0.24290324	-2.467269	0.0177
DBH.cm	0.0467329	0.00881067	5.304126	0.0000
DBH.cm.splinepoints	0.0000553	0.00000671	8.236458	0.0000

Plot of Model 2

A_densa:Model 2 : (Volume ~ dbh), Cubic spline with varFixed



7.3. Model 3- Volume with diameter at breast height (DBH) as predictor, with varPower

```
> ad.m3 <- gls(Volume.m3 ~ DBH.cm + DBH.cm.splinepoints,
  na.action=na.omit, weights = varPower(form =
  ~DBH.cm))
```

```
> summary(ad.m3)
```

Generalized least squares fit by REML

Model: Volume.m3 ~ DBH.cm + DBH.cm.splinepoints

Data: NULL

AIC	BIC	logLik
78.59912	87.40512	-34.29956

Variance function:

Structure: Power of variance covariate

Formula: ~DBH.cm

Parameter estimates:

power

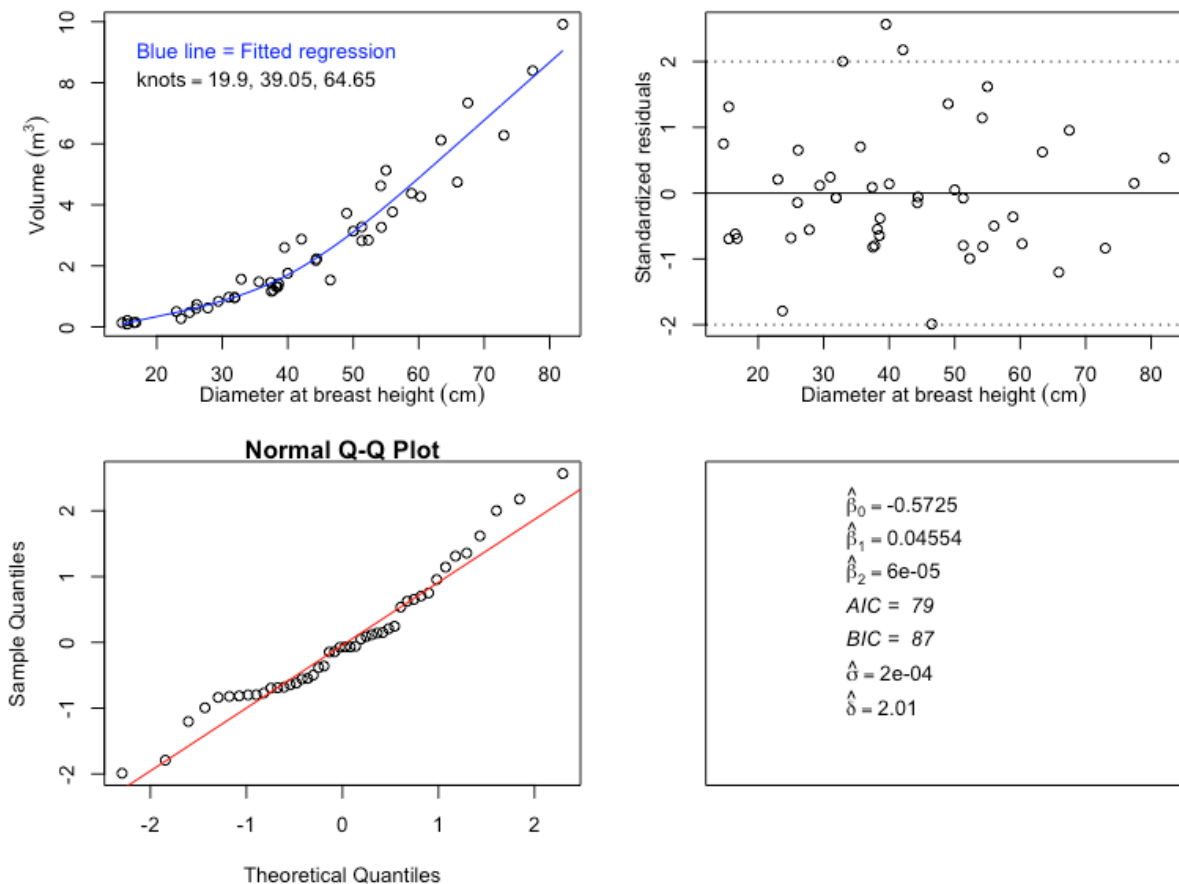
2.008802

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	-0.5724971	0.08173803	-7.004049	0
DBH.cm	0.0455388	0.00429261	10.608632	0
DBH.cm.splinepoints	0.0000562	0.00000637	8.814084	0

Plot of Model 3

A_densa:Model 3: (Volume ~ dbh), Cubic spline with varPower



7.4. Model 4 - Volume with diameter at breast height (DBH) as predictor, with varConstPower

```
> ad.m4 <- gls(Volume.m3 ~ DBH.cm + DBH.cm.splinepoints,
  na.action=na.omit, weights = varConstPower(form =
  ~DBH.cm))
```

```
> summary(ad.m4)
```

Generalized least squares fit by REML

Model: Volume.m3 ~ DBH.cm + DBH.cm.splinepoints

Data: NULL

AIC	BIC	logLik
80.59912	91.16632	-34.29956

Variance function:

Structure: Constant plus power of variance covariate

Formula: ~DBH.cm

Parameter estimates:

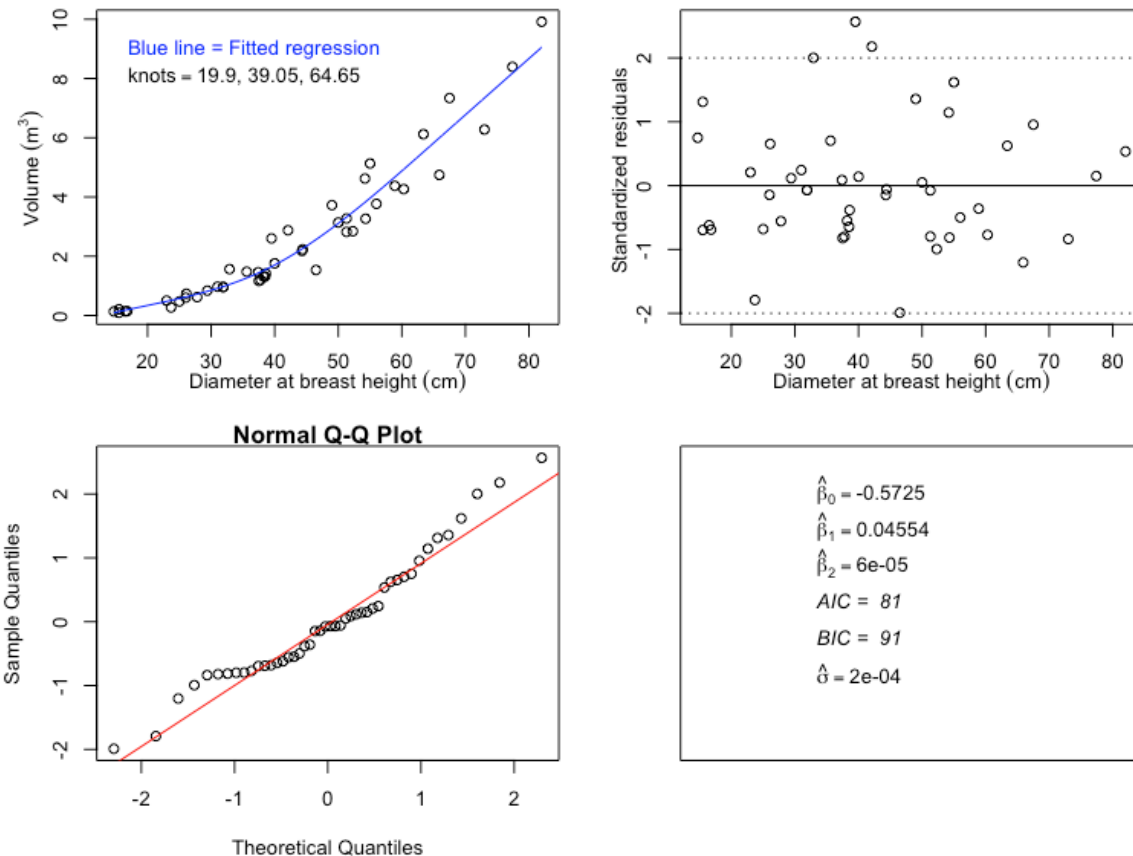
const	power
3.976635e-06	2.008800e+00

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	-0.5724972	0.08173811	-7.004043	0
DBH.cm	0.0455388	0.00429262	10.608627	0
DBH.cm.splinepoints	0.0000562	0.00000637	8.814086	0

Plot of Model 4

A_densa:Model 4: (Volume ~ dbh), Cubic spline with varConstPower



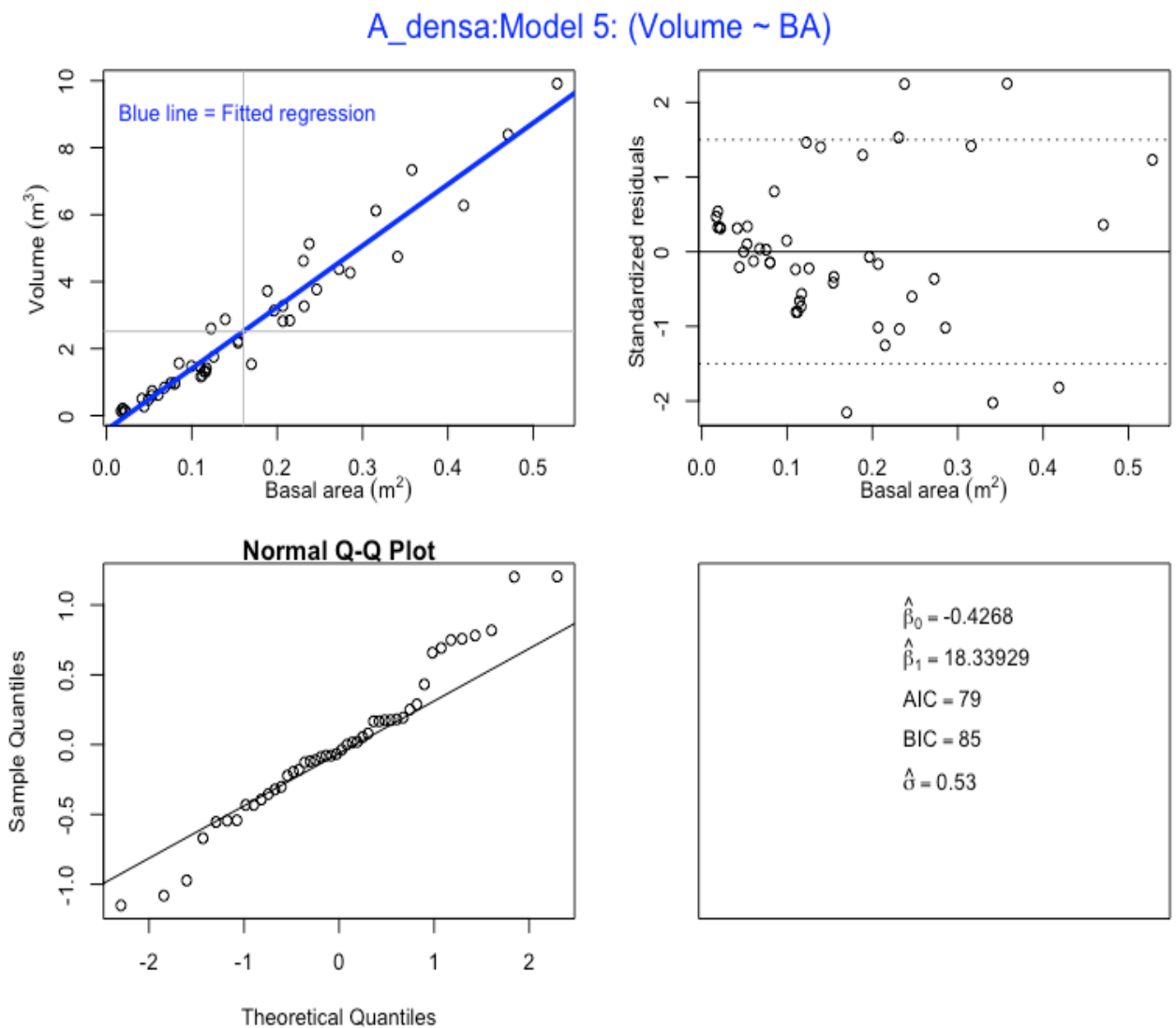
7.5. Model 5 - Volume with basal area (BA) as predictor

```
> ad.m5 <- gls(Volume.m3 ~ BA.m2)
> summary(ad.m5)
Generalized least squares fit by REML
Model: Volume.m3 ~ BA.m2
Data: NULL
      AIC      BIC    logLik
79.19238 84.54495 -36.59619
```

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	-0.426803	0.1304529	-3.271703	0.0021
BA.m2	18.339291	0.6476024	28.318753	0.0000

Plot of Model 5



7.6. Model 6 - Volume with basal area (BA) as predictor, with varFixed

```
> ad.m6<- gls(Volume.m3 ~ BA.m2 + BA.m2.splinepoints,
              na.action=na.omit, weights = varFixed(~BA.m2))
```

```
> summary(ad.m6)
```

Generalized least squares fit by REML

Model: Volume.m3 ~ BA.m2 + BA.m2.splinepoints

Data: NULL

	AIC	BIC	logLik
	45.14426	52.18907	-18.57213

Variance function:

Structure: fixed weights

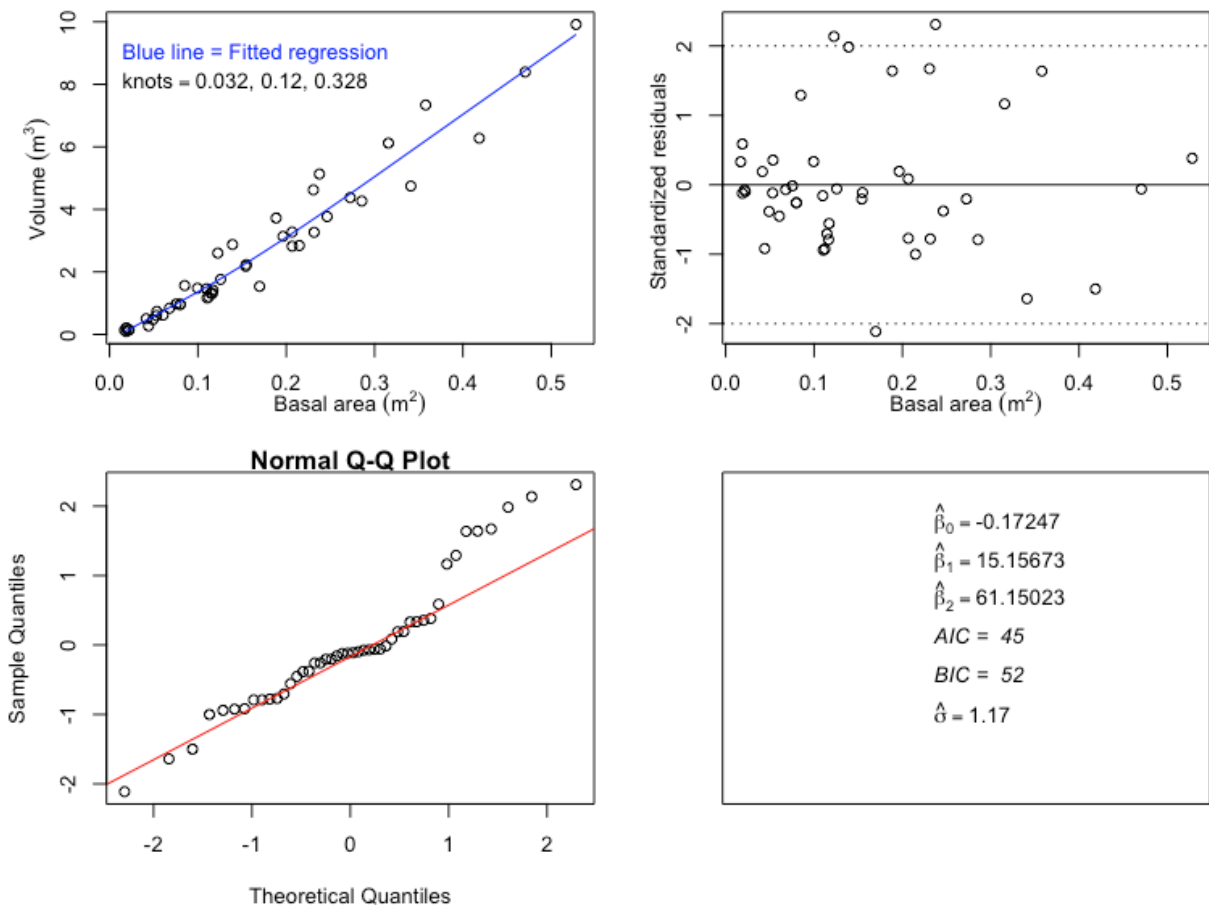
Formula: ~BA.m2

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	-0.17247	0.083239	-2.072044	0.0443
BA.m2	15.15673	1.281305	11.829143	0.0000
BA.m2.splinepoints	61.15023	31.048559	1.969503	0.0554

Plot of Model 6

A_densa:Model 6: (Volume ~ BA), Cubic spline with varFixed



7.7. Model 7 Volume with basal area (BA) as predictor, with varPower

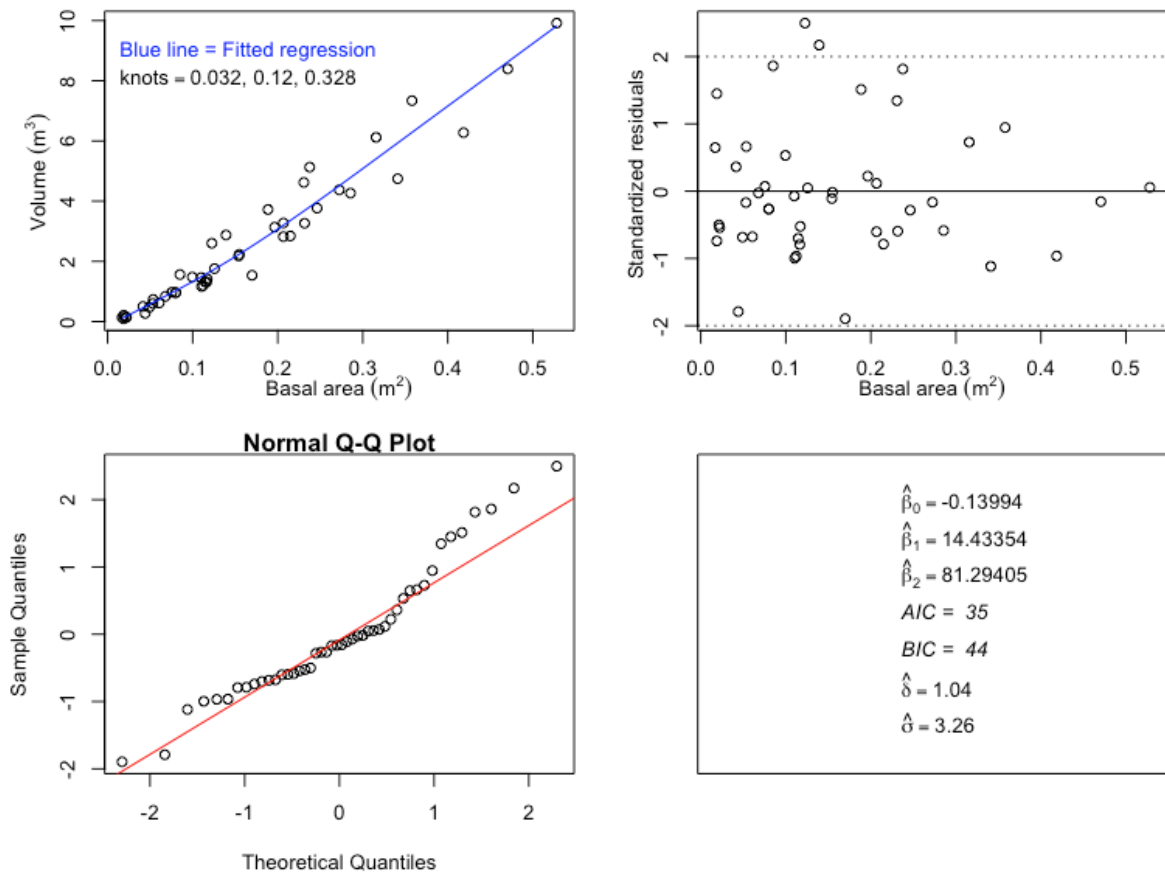
```

> ad.m7 <- gls(Volume.m3 ~ BA.m2 + BA.m2.splinepoints,
               na.action=na.omit, weights = varPower(form = ~BA.m2))
> summary(ad.m7)
Generalized least squares fit by REML
Model: Volume.m3 ~ BA.m2 + BA.m2.splinepoints
Data: NULL
      AIC      BIC    logLik
35.45481 44.26081 -12.72741
Variance function:
Structure: Power of variance covariate
Formula: ~BA.m2
Parameter estimates:
      power
1.042546
Coefficients:
                Value Std.Error  t-value p-value
(Intercept)    -0.13994  0.03475 -4.026594  0.0002
BA.m2           14.43354  0.94204 15.321639  0.0000
BA.m2.splinepoints 81.29405 33.84687  2.401819  0.0207

```

Plot of Model 7

A_densa:Model 7: (Volume ~ BA), Cubic spline with varPower



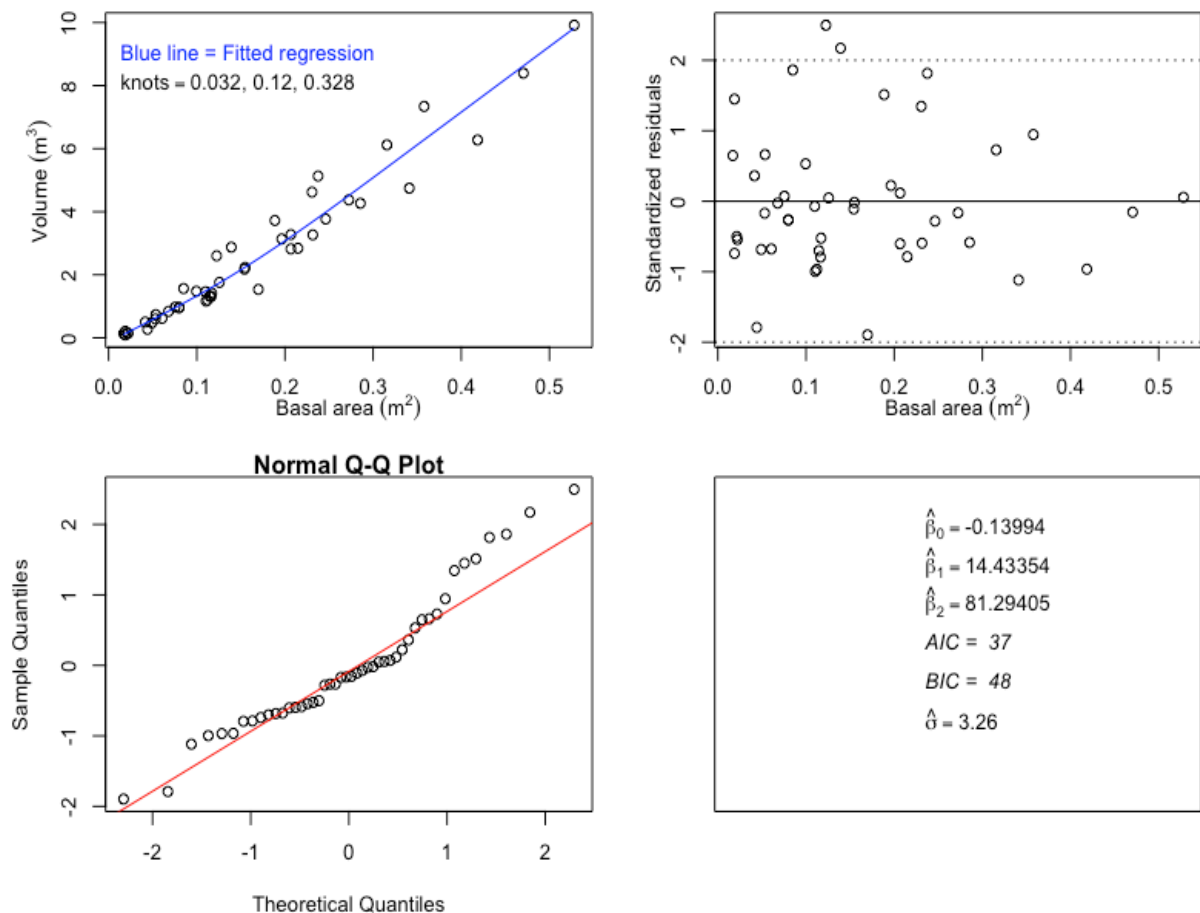
```

7.8. Model 8 – Volume with basal area (BA) as predictor, with varConstPower
> ad.m8 <- gls(Volume.m3 ~ BA.m2 + BA.m2.splinepoints,
              na.action=na.omit, weights = varConstPower(form =
              ~BA.m2))
> summary(ad.m8)
Generalized least squares fit by REML
Model: Volume.m3 ~ BA.m2 + BA.m2.splinepoints
Data: NULL
      AIC      BIC    logLik
37.45481 48.02201 -12.72741
Variance function:
Structure: Constant plus power of variance covariate
Formula: ~BA.m2
Parameter estimates:
      const      power
1.481519e-10 1.042546e+00
Coefficients:
                Value Std.Error  t-value p-value
(Intercept)    -0.13994  0.03475 -4.026593 0.0002
BA.m2           14.43354  0.94204 15.321639 0.0000
BA.m2.splinepoints 81.29405 33.84687  2.401819 0.0207

```

Plot of Model 8

A_densa:Model 8: (Volume ~ BA), Cubic spline with varConstPower



7.9. Model 9 – Volume with square of diameter at breast height * height (DBH2H) as predictor

```
> ad.m9 <- gls(Volume.m3 ~ DBH2H.m3)
> summary(ad.m9)
```

Generalized least squares fit by REML

Model: Volume.m3 ~ DBH2H.m3

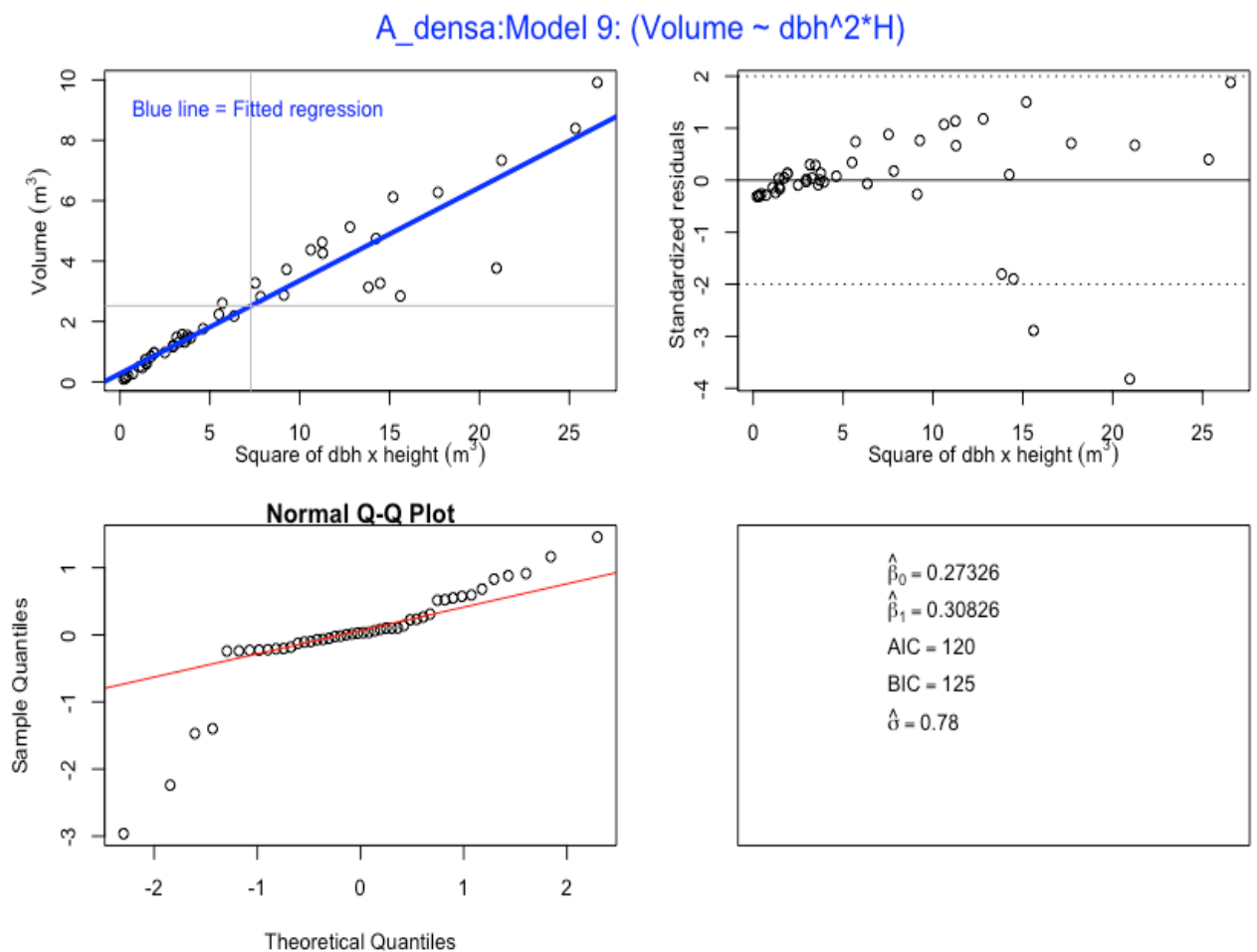
Data: NULL

	AIC	BIC	logLik
	119.9951	125.3476	-56.99754

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	0.2732628	0.16463899	1.65977	0.1041
DBH2H.m3	0.3082567	0.01628292	18.93130	0.0000

Plot of Model 9



7.10. Model 10 – Volume with square of diameter at breast height * height (DBH2H) as predictor, with varFixed

```
> ad.m10 <- gls(Volume.m3 ~ DBH2H.m3 + DBH2H.m3.splinepoints,
  na.action=na.omit, weights = varFixed(~DBH2H.m3))
> summary(ad.m10)
```

Generalized least squares fit by REML

Model: Volume.m3 ~ DBH2H.m3 + DBH2H.m3.splinepoints

Data: NULL

AIC	BIC	logLik
74.17412	81.21892	-33.08706

Variance function:

Structure: fixed weights

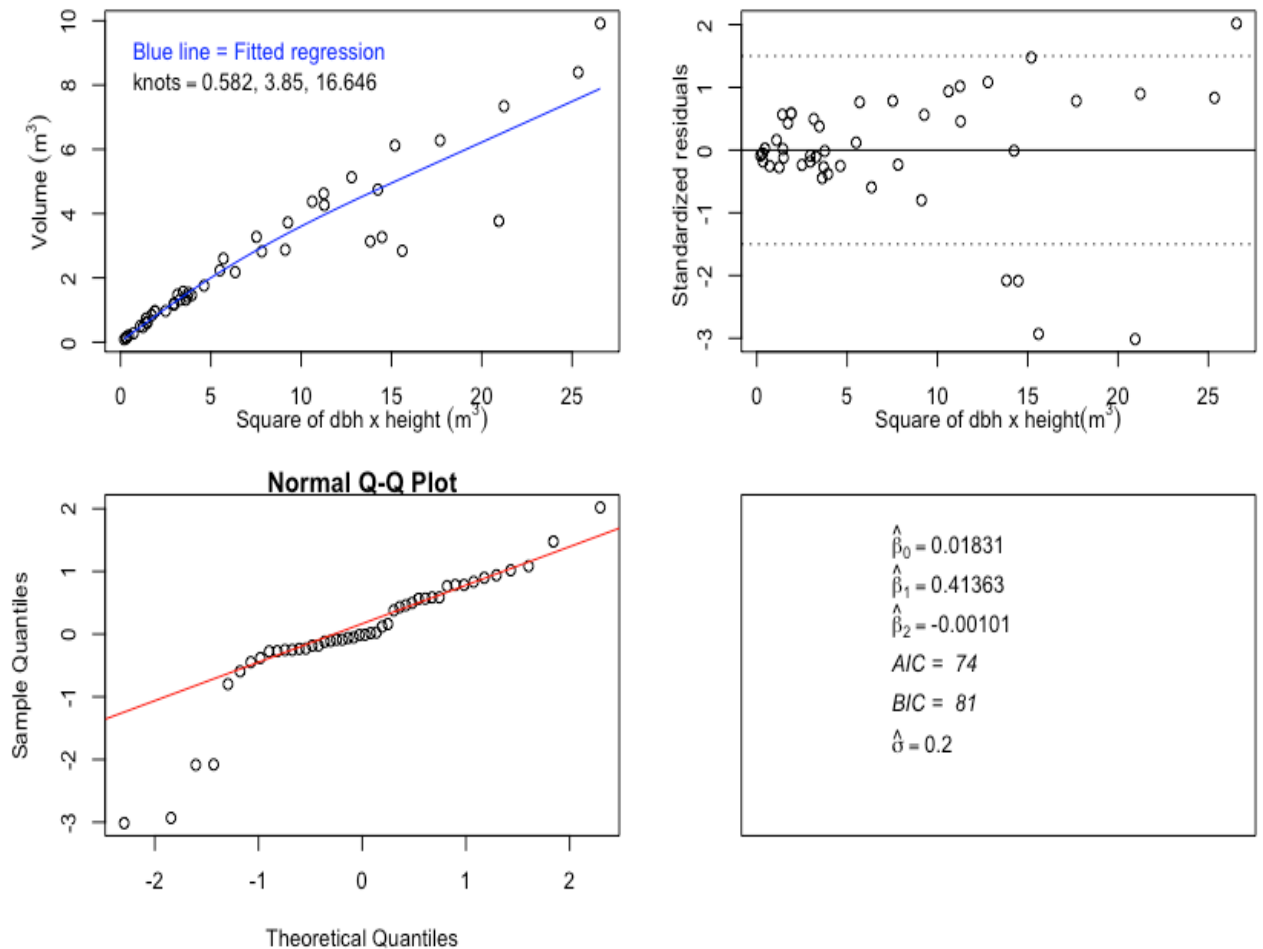
Formula: ~DBH2H.m3

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	0.0183137	0.04843170	0.378135	0.7072
DBH2H.m3	0.4136276	0.02935519	14.090442	0.0000
DBH2H.m3.splinepoints	-0.0010136	0.00033569	-3.019557	0.0042

Plot of Model 10

A_densa:Model 10: (Volume ~ dbh²*H), Cubic Spline with varFixed



7.11. Model 11– Volume with square of diameter at breast height * height (DBH2H) as predictor, with varPower

```
> ad.m11 <- gls(Volume.m3 ~ DBH2H.m3 + DBH2H.m3.splinepoints,
  na.action=na.omit, weights = varPower(form =
  ~DBH2H.m3))
```

```
> summary(ad.m11)
```

```
Generalized least squares fit by REML
Model: Volume.m3 ~ DBH2H.m3 + DBH2H.m3.splinepoints
Data: NULL
      AIC      BIC    logLik
25.3726 34.1786 -7.686299
```

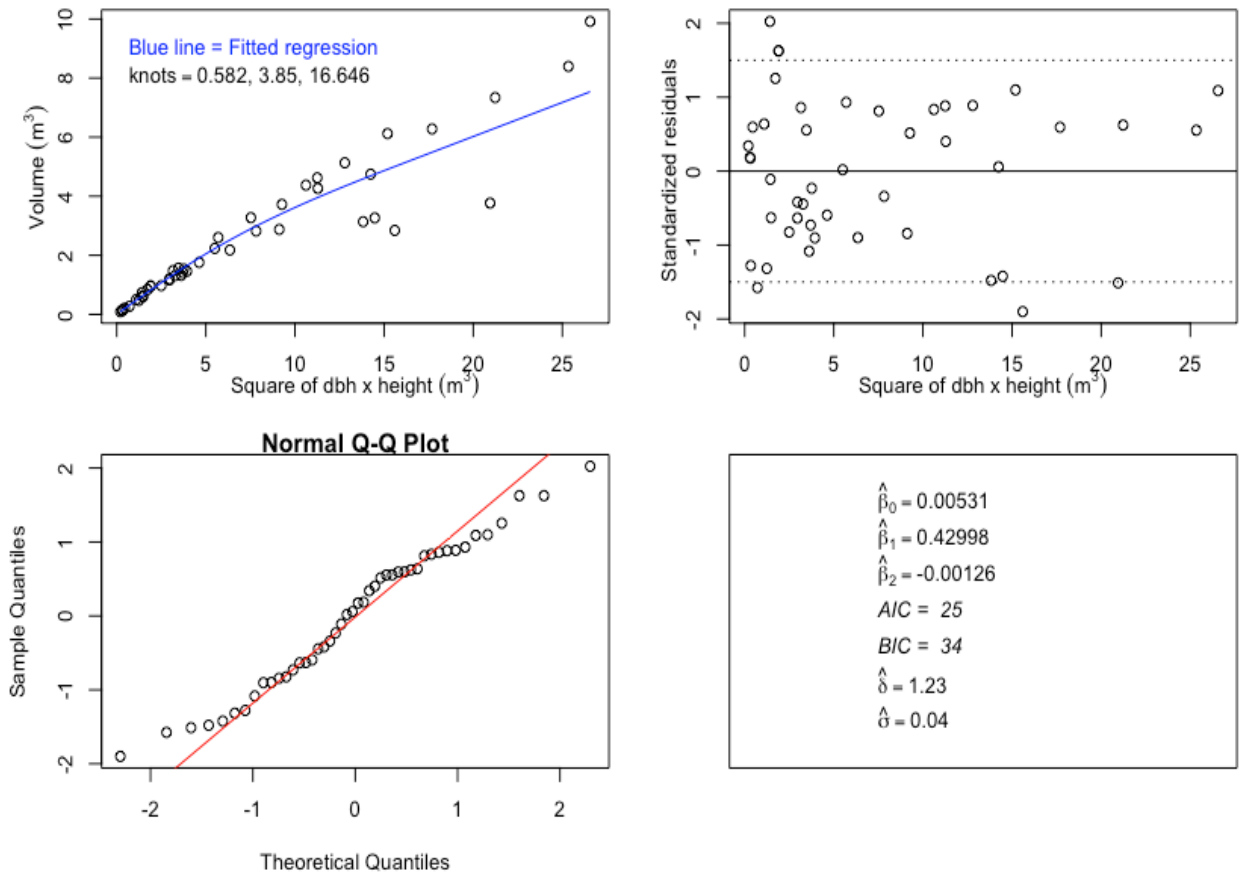
```
Variance function:
Structure: Power of variance covariate
Formula: ~DBH2H.m3
Parameter estimates:
  power
1.227666
```

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	0.0053094	0.005408648	0.98165	0.3318
DBH2H.m3	0.4299841	0.012101229	35.53227	0.0000
DBH2H.m3.splinepoints	-0.0012647	0.000272741	-4.63701	0.0000

Plot of Model 11

A_densa:Model 11: (Volume ~ dbh^2*H), Cubic Spline with varPower



7.12. Model 12 –Volume with square of diameter at breast height * height (DBH2H) as predictor, with varConstPower

```
> ad.m12 <- gls(Volume.m3 ~ DBH2H.m3 + DBH2H.m3.splinepoints,
  na.action=na.omit, weights = varConstPower(form =
  ~DBH2H.m3))
```

```
> summary(ad.m12)
```

Generalized least squares fit by REML

Model: Volume.m3 ~ DBH2H.m3 + DBH2H.m3.splinepoints

Data: NULL

AIC	BIC	logLik
27.3726	37.9398	-7.686299

Variance function:

Structure: Constant plus power of variance covariate

Formula: ~DBH2H.m3

Parameter estimates:

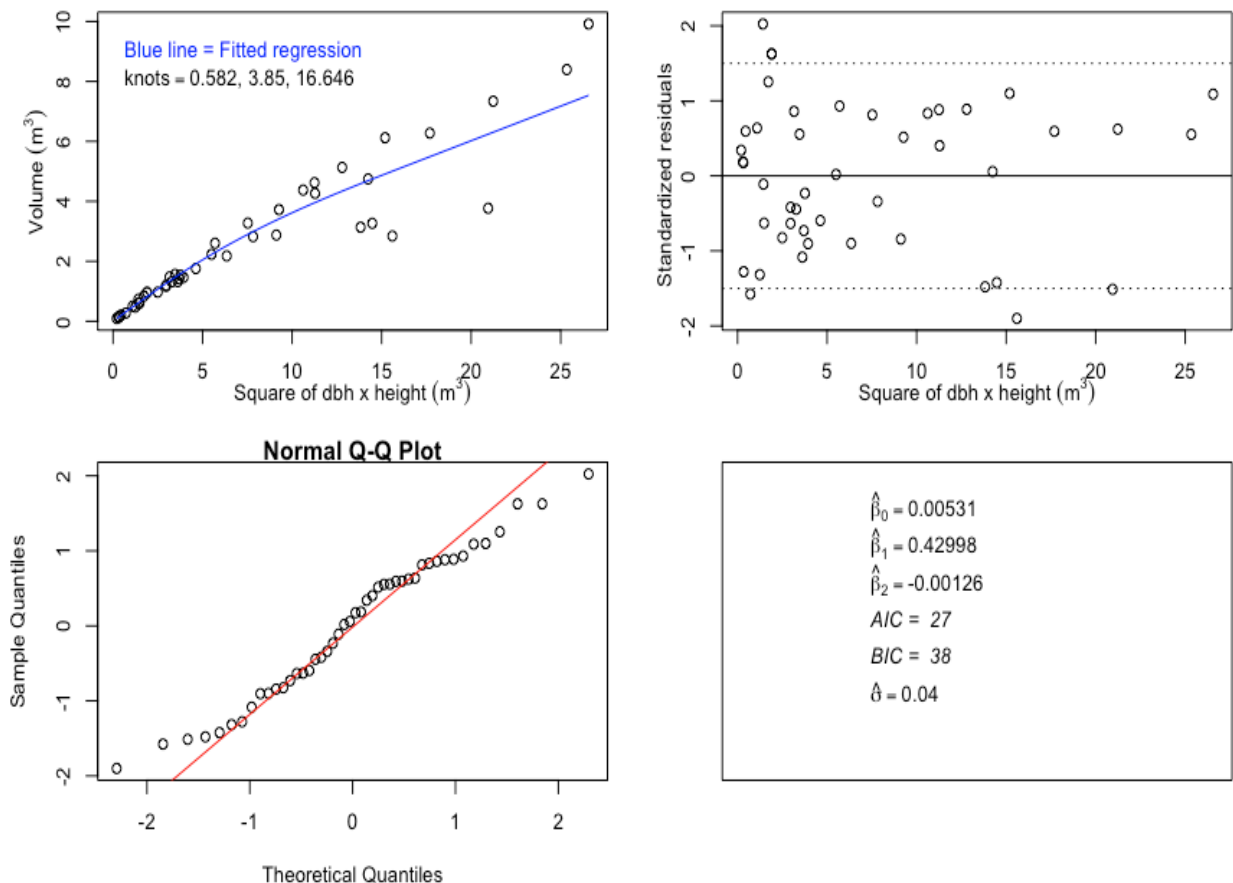
	const	power
	9.063402e-10	1.227666e+00

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	0.0053094	0.005408649	0.98165	0.3318
DBH2H.m3	0.4299841	0.012101230	35.53226	0.0000
DBH2H.m3.splinepoints	-0.0012647	0.000272741	-4.63701	0.0000

Plot of Model 12

A_densa:Model 12: (Volume ~ dbh²*H), Cubic Spline with varConstPower



7.13. Model 13 – Volume with basal area * height (BAH) as predictor

```
> ad.m13 <- gls(Volume.m3 ~ BAH.m3)
> summary(ad.m13)
```

Generalized least squares fit by REML

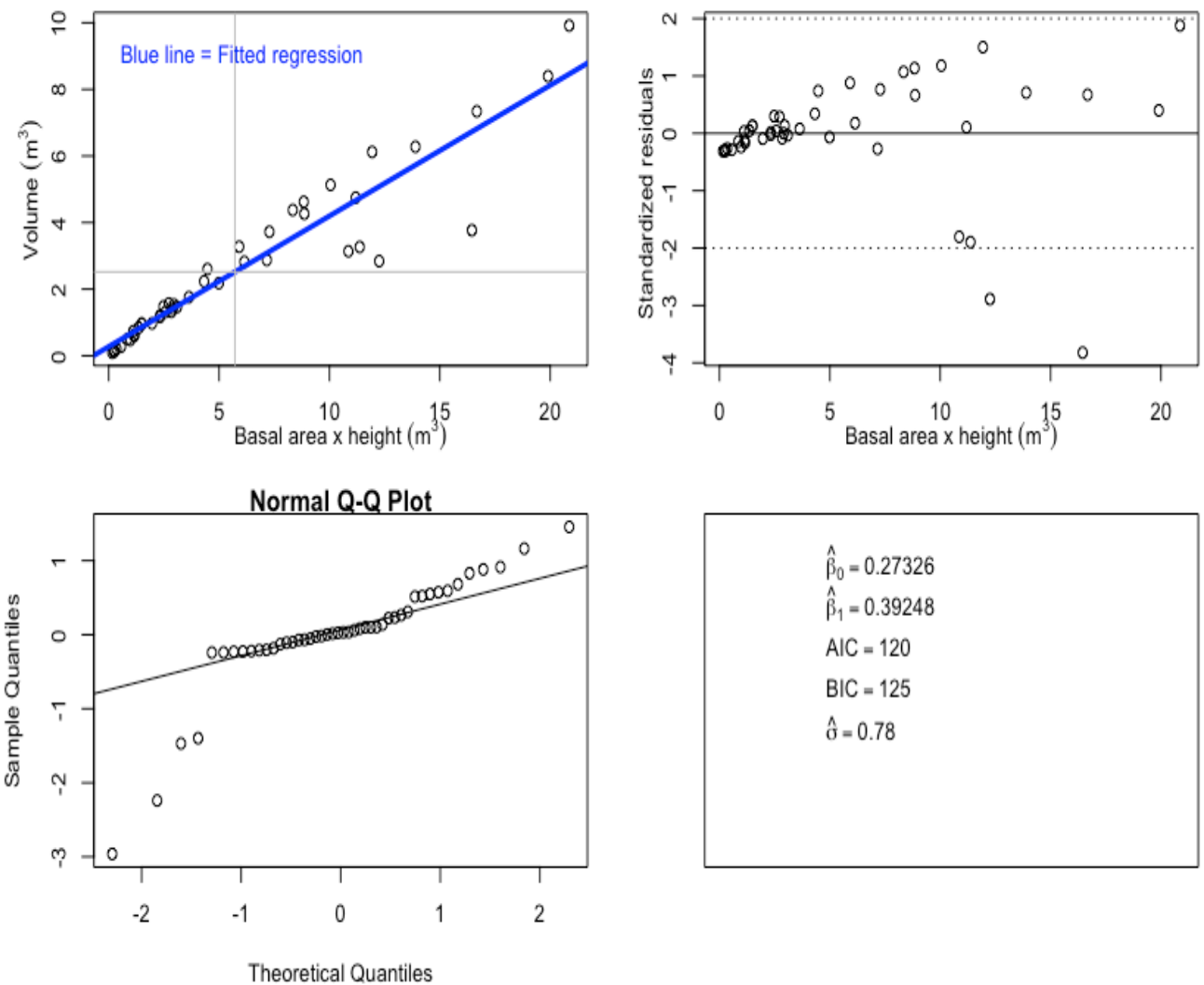
Model: Volume.m3 ~ BAH.m3

Data: NULL

	AIC	BIC	logLik
	119.5119	124.8645	-56.75597

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	0.2732628	0.16463899	1.65977	0.1041
BAH.m3	0.3924847	0.02073205	18.93130	0.0000

Plot of Model 13**A_densa:Model 13: (Volume ~ BAH)**

7.14. Model 14 – Volume with basal area * height (BAH) as predictor, with varFixed

```
> ad.m14 <- gls(Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints,
  na.action=na.omit, weights = varFixed(~BAH.m3))
```

```
> summary(ad.m14)
```

Generalized least squares fit by REML

Model: Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints

Data: NULL

	AIC	BIC	logLik
	72.2416	79.2864	-32.1208

Variance function:

Structure: fixed weights

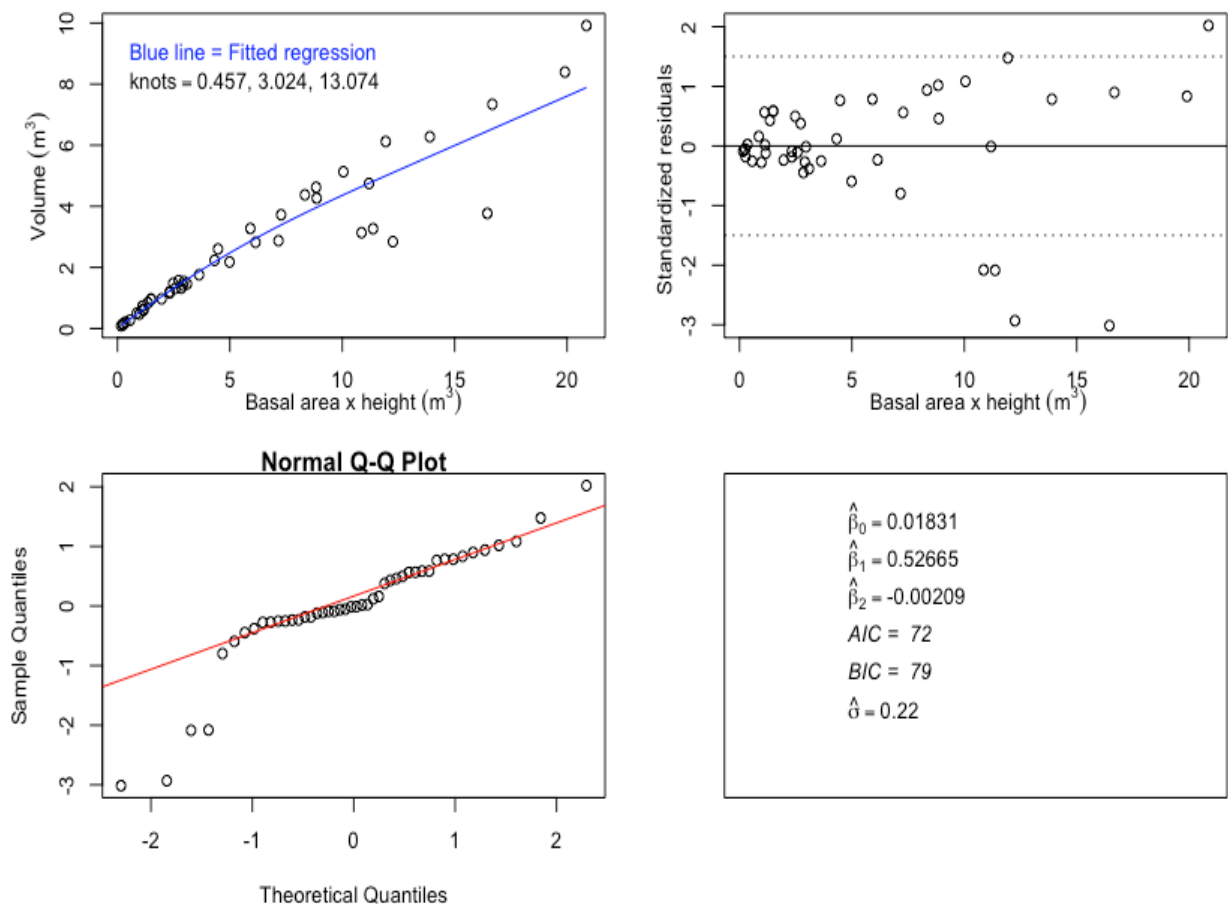
Formula: ~BAH.m3

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	0.0183137	0.04843170	0.378135	0.7072
BAH.m3	0.5266470	0.03737619	14.090442	0.0000
BAH.m3.splinepoints	-0.0020922	0.00069289	-3.019557	0.0042

Plot of Model 14

A_densa:Model 14: (Volume ~ BAH), Cubic spline with varFixed



7.15. Model 15– Volume with basal area * height (BAH) as predictor, with varPower

```
> ad.m15 <- gls(Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints,
  na.action=na.omit, weights = varPower(form =
  ~BAH.m3))
```

```
> summary(ad.m15)
```

```
Generalized least squares fit by REML
Model: Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints
Data: NULL
      AIC      BIC    logLik
23.44008 32.24608 -6.720041
```

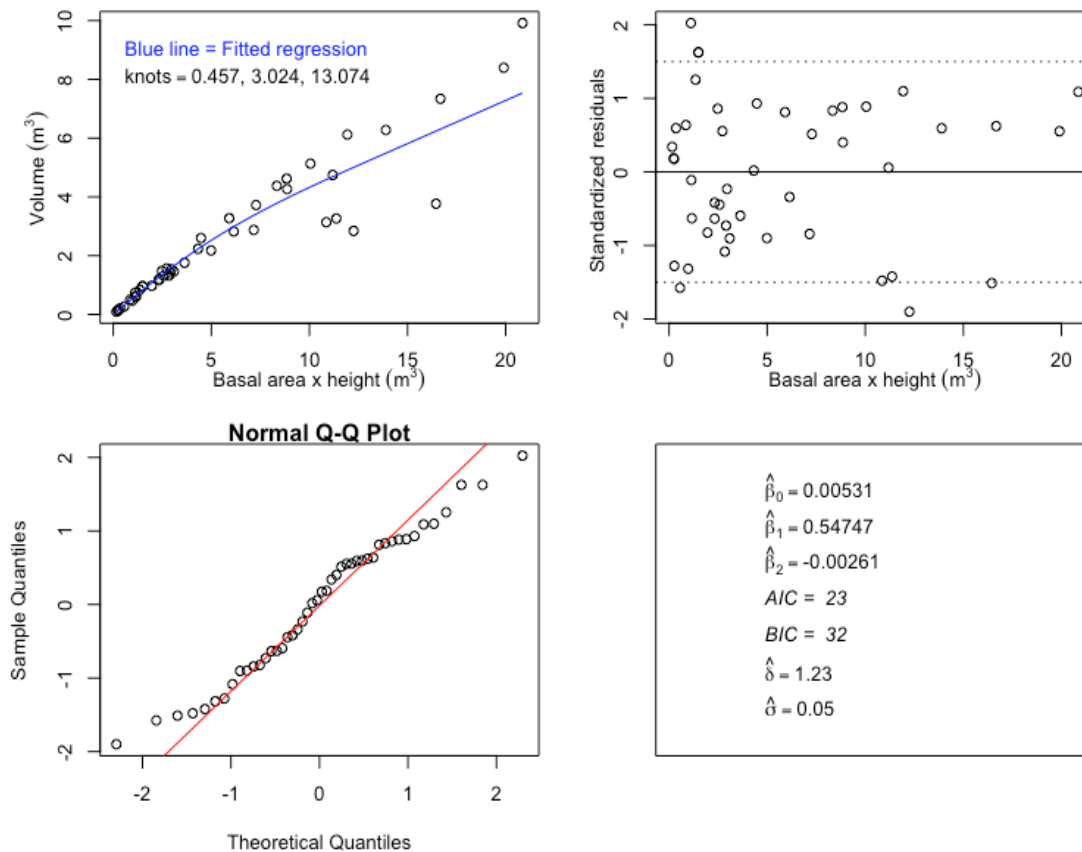
```
Variance function:
Structure: Power of variance covariate
Formula: ~BAH.m3
Parameter estimates:
      power
1.227666
```

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	0.0053094	0.005408647	0.98165	0.3318
BAH.m3	0.5474727	0.015407763	35.53227	0.0000
BAH.m3.splinepoints	-0.0026105	0.000562964	-4.63701	0.0000

Plot of Model 15

A_densa:Model 15: (Volume ~ BAH), Cubic spline with varPower



7.16. Model 16 – Volume with basal area * height (BAH) as predictor, with varConstPower

```
> ad.m16 <- gls(Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints,
  na.action=na.omit, weights = varConstPower(form =
  ~BAH.m3))
```

```
> summary(ad.m16)
```

Generalized least squares fit by REML

Model: Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints

Data: NULL

AIC BIC logLik

25.44008 36.00728 -6.720041

Variance function:

Structure: Constant plus power of variance covariate

Formula: ~BAH.m3

Parameter estimates:

const power

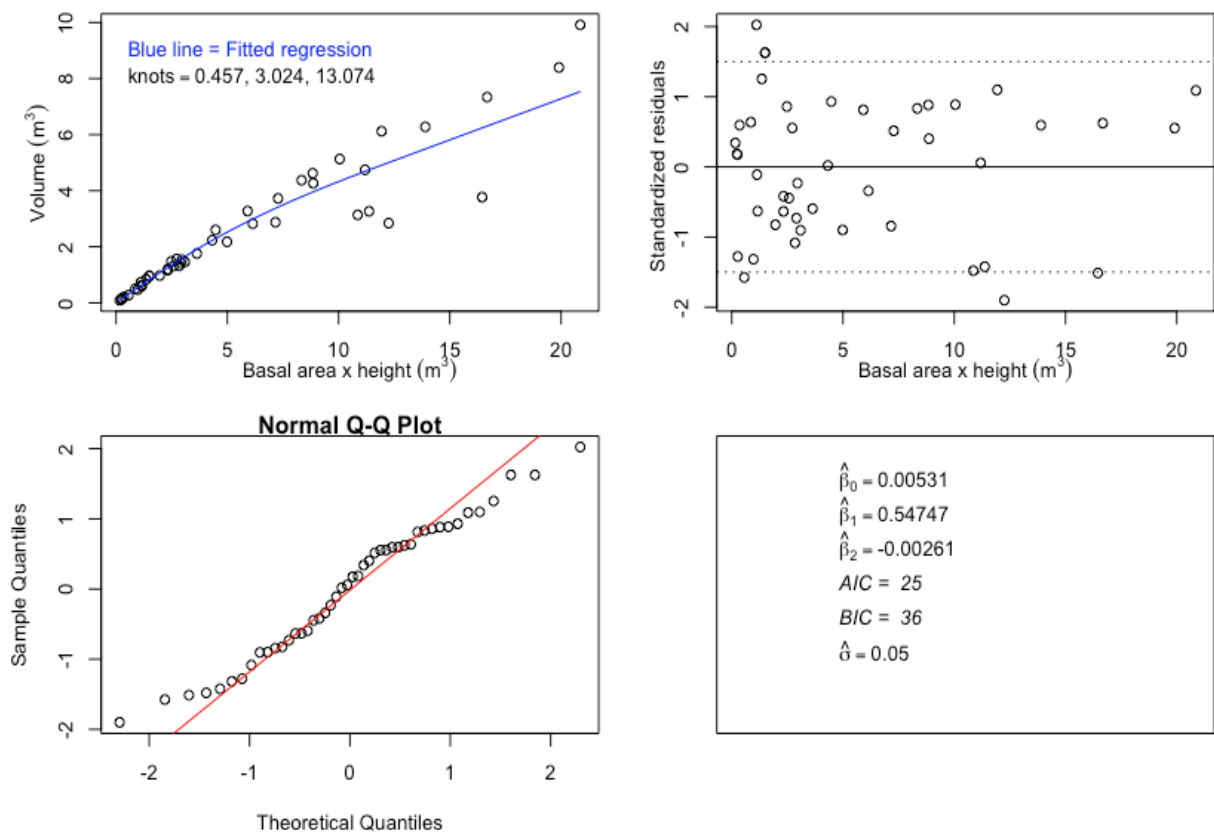
4.792722e-10 1.227666e+00

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	0.0053094	0.005408649	0.98165	0.3318
BAH.m3	0.5474727	0.015407764	35.53226	0.0000
BAH.m3.splinepoints	-0.0026105	0.000562964	-4.63701	0.0000

Plot of Model 16

A_densa:Model 16: (Volume ~ BAH), Cubic spline with varConstPower



8. Model evaluation using AIC and BIC values

SN	Model	AIC	BIC
1	Model 1 > ad.m1 <- gls(Volume.m3 ~ DBH.cm)	124	129
2	Model 2 > ad.m2 <- gls(Volume.m3 ~ DBH.cm + DBH.cm.splinepoints, na.action=na.omit, weights = varFixed(~DBH.cm))	99	106
3	Model 3 > ad.m3 <- gls(Volume.m3 ~ DBH.cm + DBH.cm.splinepoints, na.action=na.omit, weights = varPower(form = ~DBH.cm))	79	87
4	Model 4 > ad.m4 <- gls(Volume.m3 ~ DBH.cm + DBH.cm.splinepoints, na.action=na.omit, weights = varConstPower(form = ~DBH.cm))	81	91
5	Model 5 > ad.m5 <- gls(Volume.m3 ~ BA.m2)	79	85
6	Model 6 > ad.m6<- gls(Volume.m3 ~ BA.m2 + BA.m2.splinepoints, na.action=na.omit, weights = varFixed(~BA.m2))	45	52
7	Model 7 > ad.m7 <- gls(Volume.m3 ~ BA.m2 + BA.m2.splinepoints, na.action=na.omit, weights = varPower(form = ~BA.m2))	35	44
8	Model 8 > ad.m8 <- gls(Volume.m3 ~ BA.m2 + BA.m2.splinepoints, na.action=na.omit, weights = varConstPower(form = ~BA.m2))	37	48
9	Model 9 > ad.m9 <- gls(Volume.m3 ~ DBH2H.m3)	120	125
10	Model 10 > ad.m10 <-gls(Volume.m3 ~ DBH2H.m3 + DBH2H.m3.splinepoints, na.action=na.omit, weights = varFixed(~DBH2H.m3))	74	81

11	Model 11 > ad.m11 <-gls(Volume.m3 ~ DBH2H.m3 + DBH2H.m3.splinepoints, na.action=na.omit, weights = varPower(form = ~DBH2H.m3))	25	34
12	Model 12 > ad.m12 <- gls(Volume.m3 ~ DBH2H.m3 + DBH2H.m3.splinepoints, na.action=na.omit, weights = varConstPower(form = ~DBH2H.m3))	27	38
13	Model 13 > ad.m13 <- gls(Volume.m3 ~ BAH.m3)	120	125
14	Model 14 > ad.m14 <- gls(Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints, na.action=na.omit, weights = varFixed(~BAH.m3))	72	79
15	Model 15 > ad.m15 <- gls(Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints, na.action=na.omit, weights = varPower(form = ~BAH.m3))	23	32
16	Model 16 > ad.m16 <- gls(Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints, na.action=na.omit, weights = varConstPower(form = ~BAH.m3))	25	36

9. Selected Models

The best fitting models have been selected based on Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) values of the fitted models. The BIC value was mainly relied upon as it imposes a stronger penalty for the number of parameters in the model that need to be estimated. Smaller the values of AIC and BIC, better the fit of the model. Therefore, for *Abies densa*, the selected models are;

1. Model 7 (Model which doesn't use height)


```
ad.m7 <- gls(Volume.m3 ~ BA.m2 + BA.m2.splinepoints,
             na.action=na.omit, weights = varPower(form = ~BA.m2))
```
2. Model 15 (Model which uses the height)


```
ad.m15 <- gls(Volume.m3 ~ BAH.m3 + BAH.m3.splinepoints,
              na.action=na.omit, weights = varPower(form = ~BAH.m3))
```

Two models have been selected for *Abies densa*, one without height ($X_1 = BA$ which is model 7) and one with the height ($X_1 = BAH$, which is Model 15) as predictor or explanatory variable. Both the models have been fitted with natural (restricted) cubic spline function within a linear model framework. Although, nonlinear models are more flexible, they are more complicated than the linear models. The complications involved and amount of time and efforts spent on fitting nonlinear models often fail to justify by the improvements in the models. Moreover, the models fitted with natural (restricted) cubic spline functions perform well and track the curvilinearity better than nonlinear functions that were examined.

10. Demonstration of use of the selected best fit models

In general, the natural spline predictor with knots represented by t_1 , t_2 and t_3 takes the following form;

$$Y = \beta_0 + \beta_1 X + \beta_2 X_s + \varepsilon \quad (7)$$

Where X_s corresponds to value in X as follows:

$$X_s = g(X) = (X - t_1)_+^3 - (X - t_2)_+^3 \frac{(t_3 - t_1)}{(t_3 - t_2)} + (X - t_3)_+^3 \frac{(t_2 - t_1)}{(t_3 - t_2)} \quad (8)$$

and the value of the positive part functions depend on the values of the knots as follows;

$$(X - t_1)_+^3 = (X - t_1)_+^3, \text{ if } X > t_1 \text{ and } (X - t_1)_+^3 = 0, \text{ if } X < t_1 \quad (9)$$

$$(X - t_2)_+^3 = (X - t_2)_+^3, \text{ if } X > t_2, \text{ and } (X - t_2)_+^3 = 0, \text{ if } X < t_2 \quad (10)$$

$$(X - t_3)_+^3 = (X - t_3)_+^3, \text{ if } X > t_3, \text{ and } (X - t_3)_+^3 = 0, \text{ if } X < t_3 \quad (11)$$

Where t_1 , t_2 and t_3 for the above models are 10th, 50th and 90th percentiles and are called knots. The values of knots differ from species and models.

To demonstrate use of the selected models for *Abies densa* – model 7, the knots t_1 , t_2 and t_3 are 0.032, 0.12 and 0.328 as generated by the model. The model 7 has been fitted with volume as function of basal area in meter square (BA) i.e

$$BA = \pi r^2 \quad (12)$$

where in

$$r^2 = \left[\frac{dbh}{2 \times 100} \right]^2 \quad (13)$$

Where r is radius in meters and dbh is diameter at breast height in centimeters.

Therefore, *Abies densa* with diameter of 39.5 cm resulting in basal area of 0.122541748 m², the volume can be estimated using the above equation (model 7) as below. But first the value of BA.m2 has to be calculated, which is;

$$\begin{aligned} BA &= \pi r^2 = \frac{\pi * 39.5^2}{200^2} = 0.122541748 \text{ m}^2 \\ g(X) &= (X - t_1)_+^3 - (X - t_2)_+^3 \frac{(t_3 - t_1)}{(t_3 - t_2)} + (X - t_3)_+^3 \frac{(t_2 - t_1)}{(t_3 - t_2)} \\ g(BA) &= (BA - t_1)_+^3 - (BA - t_2)_+^3 \frac{(t_3 - t_1)}{(t_3 - t_2)} + (BA - t_3)_+^3 \frac{(t_2 - t_1)}{(t_3 - t_2)} \\ g(BA) &= (0.122541748 - 0.032)_+^3 - (0.122541748 - 0.12)_+^3 \frac{(0.328 - 0.032)}{(0.328 - 0.12)} + 0 \\ &= (0.090541748)_+^3 - (0.122541748 - 0.12)_+^3 \frac{(0.296)}{(0.208)} + 0 \\ &= (0.090541748)_+^3 - (0.02541748)_+^3 * 1.423076923 + 0 \\ &= 0.000742244 - 0.000000016 * 1.42410714 \\ &= 0.000742244 - 0.000000023385 \\ &= 0.0007422 \end{aligned}$$

Hence, the volume predicted for this tree by the selected model (model 7) is

$$\begin{aligned} V &= \beta_0 + \beta_1 \cdot BA + \beta_2 BA \cdot m_2 + \varepsilon \\ &= -0.13994 + 14.43354 * 0.122541748 + 81.29405 * 0.0007422 \\ &= -0.13994 + 1.76871 + 0.060336 \\ &= \mathbf{1.6891 \text{ m}^3} \end{aligned}$$

Similarly, to demonstrate model 15 with t_1 , t_2 and t_3 of 0.457, 3.024 and 13.074 respectively, we considered this same tree but with height, i.e $dbh = 39.5$ cm resulting in $BA = 0.122541748$ m² and height (H) = 36.5 m.

$$\begin{aligned} BAH &= 0.122541748 \times 36.5 \\ &= 4.47277497 \end{aligned}$$

$$g(X) = (X - t_1)_+^3 - (X - t_2)_+^3 \frac{(t_3 - t_1)}{(t_3 - t_2)} + (X - t_3)_+^3 \frac{(t_2 - t_1)}{(t_3 - t_2)}$$

$$g(BAH) = (BAH - t_1)_+^3 - (BAH - t_2)_+^3 \frac{(t_3 - t_1)}{(t_3 - t_2)} + (BAH - t_3)_+^3 \frac{(t_2 - t_1)}{(t_3 - t_2)}$$

$$\begin{aligned}
&= (4.47277497 - 0.457)_+^3 + (4.47277497 - 3.024)_+^3 \frac{(13.074 - 0.457)}{(13.074 - 3.024)} + 0 \\
&= (4.01577497)_+^3 - (1.44877497)_+^3 \frac{(12.617)}{(10.05)} + 0 \\
&= (4.01577497)_+^3 - 3.04090465 * 1.2554229 + 0 \\
&= 64.76018868 - 3.81762129 + 0 \\
&= 60.942567
\end{aligned}$$

Hence, the volume predicted by model 15 for this tree is;

$$\begin{aligned}
V &= \beta_0 + \beta_1 \cdot BAH.m3 + \beta_2 BAH.m3_2 + \varepsilon \\
&= 0.0053094 + 0.5474727 * 4.47277497 + (-0.0026105 * 60.942567) \\
&= 0.0053094 + 2.44872219 + (-0.15909057) \\
&= 2.2949410 \text{ m}^3
\end{aligned}$$

However, the field measured volume for this particular tree with DBH of 39.5 cm and height of 36.5 m is 2.602202 m³.

11. Model Performance

To assess the performance of selected models, we compared the volume predicted by selected models (7 and 15) with the volume of the tree as measured in the field. Using the equations of the selected models, volume prediction or estimation was done in R.

SN	Tree_ID	Height (in m)	DBH (in cm)	Volume in m ³ (Field measured) [A]	Predicted Volume Model_7 [B]	Predicted Volume Model_15 [C]	Difference (Field - Model_7) [A - B]	Difference (Field - Model_15) [A - C]
1	ade01	11.5	16.8	0.14659	0.180009404	0.144871904	-0.033419863	0.001717637
2	ade02	20.72	37.8	1.199397	1.521772045	1.261279706	-0.322375504	-0.061883165
3	ade03	17.4	46.5	1.536251	2.509719595	1.582357745	-0.973469045	-0.046107195
4	ade04	18.7	27.8	0.61913	0.738078807	0.625913157	-0.118949151	-0.0067835
5	ade05	32.8	65.9	4.746394	5.933461028	4.687737421	-1.187067044	0.058656563
6	ade06	33.2	73	6.27649	7.543458546	5.487882201	-1.266968567	0.788607778
7	ade07	30.6	58.9	4.375451	4.513160429	3.783981177	-0.137709482	0.591469771
8	adec01	12.7	23.7	0.271422	0.496940597	0.312034059	-0.225518761	-0.040612223
9	adec02	28.6	51.3	3.274926	3.201380375	2.896924936	0.073545638	0.378001076
10	adec03	19.9	25	0.472586	0.568970326	0.53973535	-0.096384535	-0.067149559
11	adec04	19.9	29.4	0.838287	0.843662198	0.743049921	-0.00537535	0.095236927
12	adec05	20.65	23	0.502567	0.459809162	0.474849256	0.042757864	0.02771777
13	adec07	18.75	15.5	0.207729	0.132409221	0.199003779	0.075319456	0.008724898
14	adec08	21.9	26	0.601147	0.627141626	0.640957402	-0.025994431	-0.039810207
15	adec11	12.56	16.5	0.138982	0.168684664	0.152340727	-0.029702647	-0.01335871
16	adec15	22.1	38.5	1.314582	1.589251942	1.389114198	-0.274669595	-0.074531851
17	adec16	21.04	37.5	1.166729	1.493440397	1.2605417	-0.326711423	-0.093812726
18	adec17	132.35	44.3	2.1779	2.228255883	2.517344064	-0.050356138	-0.339444318
19	adec18	48.65	32.9	1.563963	1.099199412	1.464380937	0.464763741	0.099582216
20	adec19	51.45	42.1	2.875339	1.968619501	3.371651067	0.906719154	-0.496312411
21	adec20	99.8	38.2	1.320902	1.560093647	1.525957203	-0.239191545	-0.205055102
22	adec21	57.03	52.3	2.84275	3.358967489	5.004525019	-0.516217132	-2.161774662
23	adec22	102.85	38.6	1.416755	1.599051997	1.558910855	-0.18229655	-0.142155408
24	adec23	18.7	31.9	0.960422	1.02257763	0.820623432	-0.062155159	0.139799039
25	adec24	18.7	31.9	0.960422	1.02257763	0.820623432	-0.062155159	0.139799039
26	adec25	76.9	40	1.758565	1.740610544	1.910764641	0.017954499	-0.152199598
27	adec26	31.03	60.3	4.266146	4.782336053	3.958910666	-0.51618997	0.307235417
28	adec27	78.15	37.4	1.460907	1.484073403	1.650588687	-0.023165905	-0.189681189
29	adec28	55.3	50	3.137038	3.003472094	4.588169728	0.133565591	-1.451132043
30	adec29	29.72	51.3	2.821744	3.201380375	2.987941925	-0.379635883	-0.166197433
31	adec30	49.1	54.3	3.266065	3.688010072	4.742609721	-0.42194513	-1.476544779
32	adec31	66.8	56	3.768954	3.982047467	6.239320204	-0.213093391	-2.470366128
33	adec32	52.65	31	0.9717	0.956137714	1.070761696	0.015562443	-0.099061538
34	adw01	24.9	35.6	1.478183	1.32179313	1.340656598	0.156390181	0.137526713

35	adw02	37.8	63.4	6.120486	5.405805596	4.9103462	0.714680612	1.210140009
36	adw03	20.8	26.1	0.734994	0.633092989	0.613824609	0.101900567	0.121168946
37	adw04	38.3	54.2	4.62264	3.671121766	3.950725357	0.951518385	0.671914794
38	adw05	27.9	44.4	2.23293	2.240572312	2.226933292	-0.007642256	0.005996764
39	adw06	8.4	15.5	0.093949	0.132409221	0.092084482	-0.038460286	0.001864454
40	adw08	39.5	82	9.916287	9.820855941	7.53425267	0.095430918	2.38203419
41	adwc01	38.6	49	3.723356	2.856592696	3.414002998	0.866763446	0.309353144
42	adwc02	42.3	55	5.131366	3.807502761	4.339459806	1.323863377	0.791906332
43	adwc03	13.8	14.7	0.135164	0.105021262	0.133532451	0.030142544	0.001631355
44	adwc04	46.6	67.5	7.33949	6.281910992	6.304777373	1.057578895	1.034712514
45	adwc05	42.3	77.4	8.394678	8.623810159	7.252968762	-0.229132101	1.141709296
46	adwc06	36.5	39.5	2.602202	1.68910934	2.294940508	0.913093045	0.307261877
				115.784	115.8083594	114.824163	-0.024401641	0.959794777

From the above table, the difference [A-B] provides difference between the volume measured in the field (actual volume) and the volume predicted by model 7. The figures with negative (-) indicates that the volume has been over-predicted by the model 7 vis-à-vis actual volume of the particular tree. And the figures without negative (-) sign indicates the under prediction of volume by the model 7.

Similarly, the difference [A-C] is the difference between the actual volume and the volume predicted by the model 15. Same explanation is applicable here – the figures with negative sign indicates overprediction of volume by the model and vice-versa, while those figures without (-) are under prediction of volume by the model 15.

Summation of the figures in the difference column result in -0.024401641 and 0.959794777 for model 7 and model 15 respectively. These indicate that the model 7 over predicts total volume for 46 trees by 0.024401641 m³, while the model 15 under predicts the total volume of 46 trees by 0.959794777 m³.

12. Limitations of the model

The model has the following limitations;

1. The modeling has been done based on only 46 sample trees. The model can be further improved by increasing the samples.
2. The diameter for the sample trees ranges between minimum of 14.7 cm to 82 cm (over bark). Thus, the model prediction for trees above 82 cm should be done with caution.

13. Conclusion

The model 7 which doesn't use the height as predictor slightly over predicts but model 15 under predicts a bit, as empirically shown above. Like in other conifer species that we have modelled (*Pinus wallichiana*, *Juniperus recurva*) for which the model with height as predictor was observed to have lowest AIC and BIC values for *Abies densa*.

This, therefore, leads us to conclude that the best model for *Abies densa*, out of 16 models fitted above, is model 15. But since the two models are fitted with different predictors (one with and other without height as predictor), it leads us to conclude two best fit models for *Abies densa*, namely;

1. Model 7: the best fit model that doesn't use height
2. Model 15: the best fit model which uses height as predictor.

14. Acknowledgement

We would like to express our appreciation to the biomass equation development team led by Mr. Yograj Chettri, Research Officer at UWICER, formerly RDC who collected volume data (diameter and height) as part of field work for biomass equation development exercise.

Immense gratitude is also due to Professor Timothy Gordon Gregoire, School of Forestry and Environmental Studies (FES), Yale University who has been a guide and mentor as we worked on this assignment.

Thanks are also due to our Director, Mr. Lobzang Dorji and other colleagues working in FRMD for their support and advice.

Had it not been for the immense support, guidance and encouragement that the abovementioned people have generously provided to us, this task would have remained incomplete. Thanks a lot!

15. References

1. Lee, D., Seo, Y., & Choi, J. (2017). Estimation and validation of stem volume equations for *Pinus densiflora*, *Pinus koraiensis*, and *Larix kaempferi* in South Korea. *Forest Science and Technology*, 13(2), 77-82.
2. Umunay, P., Gregoire, T., & Ashton, M. (2017). Estimating biomass and carbon for *Gilbertiodendron dewevrei* (De Wild) Leonard, a dominant canopy tree of African tropical Rainforest: Implications for policies on carbon sequestration. *Forest Ecology and Management*, 404, 31-44.
3. White, J. C., Coops, N. C., Wulder, M. A., Vastaranta, M., Hilker, T., & Tompalski, P. (2016). Remote sensing technologies for enhancing forest inventories: A review. *Canadian Journal of Remote Sensing*, 42(5), 619-641.
4. Mohammadi, J., Shataee, S., & Babanezhad, M. (2011). Estimation of forest stand volume, tree density and biodiversity using Landsat ETM+ Data, comparison of linear and regression tree analyses. *Procedia Environmental Sciences*, 7, 299-304.
5. Fagan, M., & DeFries, R. (2009). Measurement and Monitoring of the World's Forests. *Resources for the Future*, 129.
6. Feng, Z. K., Yang, B.G., Luo, X., Han, G.S., Guo, X.X., (2008). Experiment of estimating forest stand volume with LiDAR technology. *The International Archives of the Photogrammetry, Remote Sensing and Spatial Information Sciences.*, XXXVII.
7. McRoberts, R. E., & Tomppo, E. O. (2007). Remote sensing support for national forest inventories. *Remote Sensing of Environment*, 110(4), 412-419.
8. Westfall, J. A., & Patterson, P. L. (2007). Measurement variability error for estimates of volume change. *Canadian Journal of Forest Research*, 37(11), 2201-2210.
9. Lu, D. (2006). The potential and challenge of remote sensing-based biomass estimation. *International Journal of Remote Sensing*, 27(7), 1297-1328.
10. Sadiq, R. A. (2006). A new approach to log volume estimation. *Southern Journal of Applied Forestry*, 30(1), 30-39.
11. Hyypä, J., Mielonen, T., Hyypä, H., Maltamo, M., Yu, X., Honkavaara, E., & Kaartinen, H. (2005). Using individual tree crown approach for forest volume extraction with aerial images and laser point clouds.
12. Patterson, D. W., & Doruska, P. F. (2004). A new and improved modification to Smalian's equation for butt logs. *Forest Products Journal*, 54(4), 69.
13. Eerikäinen, K. (2001). Stem volume models with random coefficients for *Pinus kesiya* in Tanzania, Zambia, and Zimbabwe. *Canadian Journal of Forest Research*, 31(5), 879-888.
14. Gregoire, T. G., & Schabenberger, O. (1996). Nonlinear mixed-effects modeling of cumulative bole volume with spatially correlated within-tree data. *Journal of Agricultural, Biological, and Environmental Statistics*, 107-119.
15. Bi, H. (1994). Volume equations for six *Eucalyptus* species on the south-east tablelands of New South Wales: Research Division State Forests of New South Wales.
16. Laumans, P. (1994). Height-diameter functions from PIS for country-level site classification and local volume table selection. Thimphu.
17. Biging, G. S. (1988). Estimating the accuracy of volume equations using taper equations of stem profile. *Canadian Journal of Forest Research*, 18(8), 1002-1007.
18. Reed, D. D., & Byrne, J. C. (1985). A simple, variable form volume estimation system. *The Forestry Chronicle*, 61(2), 87-90.
19. Avery, T.E., and Burkhart, H.E. (1983). *Forest Measurements*, Third Edition. McGraw-Hill, Inc.
20. Sadiq, R. A., & Smith, V. G. (1983). Estimation of individual tree volumes with age and diameter. *Canadian Journal of Forest Research*, 13(1), 32-39.
21. Cochran, P. (1982). Estimating wood volumes for Douglas-fir and white fir from outside bark measurements. *Forest Science*, 28(1), 172-174.

22. Bredenkamp, B. (1982). Volume regression equations for *Eucalyptus grandis* on the coastal plain of Zululand. *South African Forestry Journal*, 122(1), 66-69.
23. Cao, Q. v., & Burkhart, H. E. (1980). Cubic-foot volume of loblolly pine to any height limit. *Southern Journal of Applied Forestry*, 4(4), 166-168.
24. Cao, Q. V., Burkhart, H. E., & Max, T. A. (1980). Evaluation of two methods for cubic-volume prediction of loblolly pine to any merchantable limit. *Forest Science*, 26(1), 71-80.
25. Goulding, C. (1979). Cubic spline curves and calculation of volume of sectionally measured trees. *NZJ For. Sci*, 9(1), 89-99.
26. Burkhart, H. E. (1977). Cubic-foot volume of loblolly pine to any merchantable top limit. *Southern Journal of Applied Forestry*, 1(2), 7-9.
27. Heger, L. (1965). A trial of Hohenadl's method of stem form and stem volume estimation. *The Forestry Chronicle*, 41(4), 466-475.

16. Annexure – Dataset for *Abies densa*

SN	Tree_ID	Height.m	DBH.cm	Volume.m3	BA.m2	BAH.m3	DBH2H.m3
1	ade01	11.5	16.8	0.14659	0.022167	0.254921	0.324576
2	ade02	20.72	37.8	1.199397	0.112221	2.325216	2.960556
3	ade03	17.4	46.5	1.536251	0.169823	2.954915	3.762315
4	ade04	18.7	27.8	0.61913	0.060699	1.135066	1.445211
5	ade05	32.8	65.9	4.746394	0.341083	11.18754	14.24442
6	ade06	33.2	73	6.27649	0.418539	13.89548	17.69228
7	ade07	30.6	58.9	4.375451	0.272471	8.337616	10.61578
8	adec01	12.7	23.7	0.271422	0.044115	0.560261	0.713346
9	adec02	28.6	51.3	3.274926	0.206692	5.911404	7.526633
10	adec03	19.9	25	0.472586	0.049087	0.976839	1.24375
11	adec04	19.9	29.4	0.838287	0.067887	1.350945	1.720076
12	adec05	20.65	23	0.502567	0.041548	0.857957	1.092385
13	adec07	18.75	15.5	0.207729	0.018869	0.353797	0.450469
14	adec08	21.9	26	0.601147	0.053093	1.162735	1.48044
15	adec11	12.56	16.5	0.138982	0.021382	0.268564	0.341946
16	adec15	22.1	38.5	1.314582	0.116416	2.572786	3.275773
17	adec16	21.04	37.5	1.166729	0.110447	2.323797	2.95875
18	adec17	32.35	44.3	2.1779	0.154134	4.986222	6.348655
19	adec18	32	32.9	1.563963	0.085012	2.720393	3.463712
20	adec19	51.45	42.1	2.875339	0.139205	7.162085	9.119049
21	adec20	24.8	38.2	1.320902	0.114608	2.842289	3.618915
22	adec21	57.03	52.3	2.84275	0.214829	12.25171	15.59936
23	adec22	24.85	38.6	1.416755	0.117021	2.907976	3.702551
24	adec23	18.7	31.9	0.960422	0.079923	1.494558	1.902931
25	adec24	18.7	31.9	0.960422	0.079923	1.494558	1.902931
26	adec25	28.9	40	1.758565	0.125664	3.631681	4.624
27	adec26	31.03	60.3	4.266146	0.285578	8.86148	11.28279
28	adec27	28.15	37.4	1.460907	0.109858	3.092513	3.937509
29	adec28	55.3	50	3.137038	0.19635	10.85813	13.825
30	adec29	29.72	51.3	2.821744	0.206692	6.1429	7.821383
31	adec30	49.1	54.3	3.266065	0.231574	11.37028	14.47709
32	adec31	66.8	56	3.768954	0.246301	16.4529	20.94848
33	adec32	26	31	0.9717	0.075477	1.962396	2.4986
34	adw01	24.9	35.6	1.478183	0.099538	2.478502	3.155726
35	adw02	37.8	63.4	6.120486	0.315696	11.93329	15.19394
36	adw03	20.8	26.1	0.734994	0.053502	1.112844	1.416917
37	adw04	38.3	54.2	4.62264	0.230722	8.836641	11.25116
38	adw05	27.9	44.4	2.23293	0.15483	4.319764	5.500094
39	adw06	8.4	15.5	0.093949	0.018869	0.158501	0.20181

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40	adw08	39.5	82	9.916287	0.528102	20.86002	26.5598
41	adwc01	38.6	49	3.723356	0.188574	7.27896	9.26786
42	adwc02	42.3	55	5.131366	0.237583	10.04976	12.79575
43	adwc03	13.8	14.7	0.135164	0.016972	0.234209	0.298204
44	adwc04	46.6	67.5	7.33949	0.357847	16.67567	21.23213
45	adwc05	42.3	77.4	8.394678	0.470513	19.90271	25.34091
46	adwc06	36.5	39.5	2.602202	0.122542	4.472774	5.694913